Nơi khởi đầu ước mơ

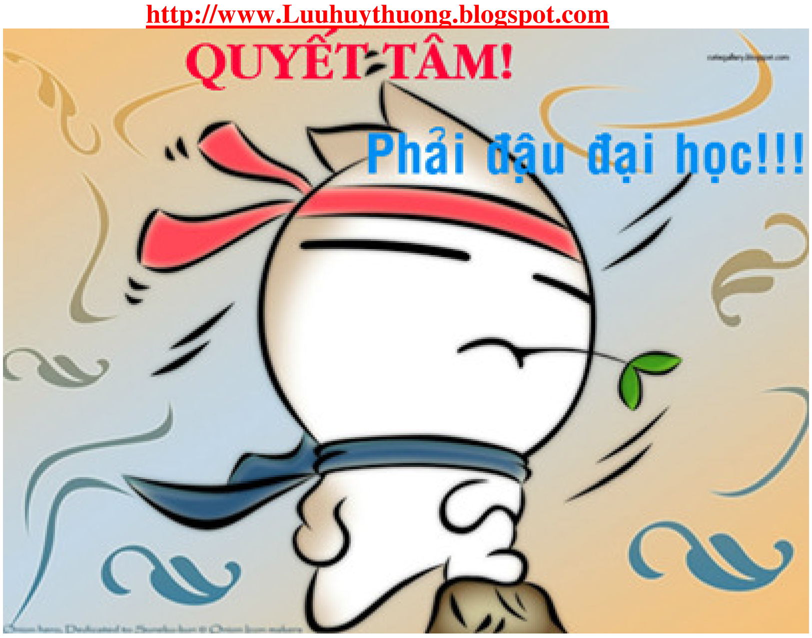
TUYỂN TẬP TÍCH PHÂN

(ĐÁP ÁN CHI TIẾT)

BIÊN SOẠN: LƯU HUY THƯỞNG

Toàn bộ tài liệu của thầy ở trang:

http://www.Luuhuythuong.blogspot.com



HỌ VÀ TÊN: ……………………………………………………………

LỚP :……………………………………………………………

TRƯỜNG :……………………………………………………………

HÀ NỘI, 4/2014

TÍCH PHÂN CƠ BẢN

Toàn bộ tài liệu luyện thi đại học môn toán của thầy Lưu Huy Thưởng:

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HT 1.Tính các tích phân sau:

a) $I_1 = \int_0^1 x^3 dx$ b) $I_2 = \int_0^1 (2x+1)^3 dx$ c) $I_3 = \int_0^1 (1-4x)^3 dx$

d) $I_4 = \int_0^1 (x-1)(x^2-2x+5)^3 dx$ e) $I_5 = \int_0^1 (2x-3)(x^2-3x+1)^3 dx$

Bài giải

a) $I_1 = \int_0^1 x^3 dx = \frac{x^4}{4}\Big|_0^1 = \frac{1}{4}$

b) $I_2 = \int_0^1 (2x+1)^3 dx$ **Chú ý:** $d(2x+1) = 2dx \Rightarrow dx = \frac{1}{2}d(2x+1)$

$\Rightarrow I_2 = \int_0^1 (2x+1)^3 dx = \frac{1}{2}\int_0^1 (2x+1)^3 d(2x+1) = \frac{1}{2}\frac{(2x+1)^4}{4}\Big|_0^1 = \frac{81}{8} - \frac{1}{8} = 10$

c) $I_3 = \int_0^1 (1-4x)^3 dx$ **Chú ý:** $d(1-4x) = -4dx \Rightarrow dx = -\frac{1}{4}d(1-4x)$

$\Rightarrow I_3 = \int_0^1 (1-4x)^3 dx = -\frac{1}{4}\int_0^1 (1-4x)^3 d(1-4x) = -\frac{1}{4}\frac{(1-4x)^4}{4}\Big|_0^1 = -\frac{81}{16} + \frac{1}{16} = -5$

d) $I_4 = \int_0^1 (x-1)(x^2-2x+5)^3 dx$ **Chú ý:** $d(x^2-2x+5) = (2x-2)dx \Rightarrow (x-1)dx = \frac{1}{2}d(x^2-2x+5)$

$\Rightarrow I_4 = \int_0^1 (x-1)(x^2-2x+5)^3 dx = \frac{1}{2}\int_0^1 (x^2-2x+5)^3 d(x^2-2x+5)$

$= \frac{1}{2}.\frac{(x^2-2x+5)^4}{4}\Big|_0^1 = 162 - \frac{615}{8} = \frac{671}{8}$

e) $I_5=\int_0^1 (2x-3)(x^2-3x+1)^3dx$ **Chú ý:** $d(x^2-3x+1)=(2x-3)dx$

$\Rightarrow I_5=\int_0^1 (2x-3)(x^2-3x+1)^3dx=\int_0^1 (x^2-3x+1)^3d(x^2-3x+1)$

$=\left.\frac{(x^2-3x+1)^4}{4}\right|_0^1=\frac{1}{4}-\frac{1}{4}=0$

HT 2.Tính các tích phân sau:

a) $I_1=\int_0^1 \sqrt{x}dx$ **b)** $I_2=\int_2^7 \sqrt{x+2}dx$ **c)** $I_3=\int_0^4 \sqrt{2x+1}dx$

d) $I_4=\int_0^1 x\sqrt{1+x^2}dx$ **e)** $I_5=\int_0^1 x\sqrt{1-x^2}dx$ **f)** $I_6=\int_0^1 (1-x)\sqrt{x^2-2x+3}dx$

g) $I_7=\int_0^1 x^2\sqrt{x^3+1}dx$ **h)** $I_8=\int_0^1 (x^2-2x)\sqrt{x^3-3x^2+2}dx$

Bài giải

a) $I_1=\int_0^1 \sqrt{x}dx=\left.\frac{2}{3}x\sqrt{x}\right|_0^1=\frac{2}{3}$

b) $I_2=\int_2^7 \sqrt{x+2}dx=\left.\frac{2}{3}(x+2)\sqrt{x+2}\right|_2^7=18-\frac{16}{3}=\frac{38}{3}$

c) $I_3=\int_0^4 \sqrt{2x+1}dx=\frac{1}{2}\int_0^4 \sqrt{2x+1}d(2x+1)=\left.\frac{1}{2}.\frac{2}{3}(2x+1)\sqrt{2x+1}\right|_0^4=9-\frac{1}{3}=\frac{26}{3}$

d) $I_4=\int_0^1 x\sqrt{1+x^2}dx=\frac{1}{2}\int_0^1 \sqrt{1+x^2}d(1+x^2)=\left.\frac{1}{2}.\frac{2}{3}(1+x^2)\sqrt{1+x^2}\right|_0^1=\frac{2\sqrt{2}}{3}-\frac{1}{3}$

e) $I_5=\int_0^1 x\sqrt{1-x^2}dx=-\frac{1}{2}\int_0^1 \sqrt{1-x^2}d(1-x^2)=\left.-\frac{1}{2}.\frac{2}{3}(1-x^2)\sqrt{1-x^2}\right|_0^1=0+\frac{1}{3}=\frac{1}{3}$

f) $I_6=\int_0^1 (1-x)\sqrt{x^2-2x+3}dx=-\frac{1}{2}\int_0^1 \sqrt{x^2-2x+3}d(x^2-2x+3)$

$=\left.-\frac{1}{2}.\frac{2}{3}(x^2-2x+3)\sqrt{x^2-2x+3}\right|_0^1=-\frac{2\sqrt{2}}{3}+\sqrt{3}$

g) $I_7 = \int\limits_0^1 x^2\sqrt{x^3+1}dx = \frac{1}{3}\int\limits_0^1 \sqrt{x^3+1}d(x^3+1) = \frac{1}{3}.\frac{2}{3}(x^3+1)\sqrt{x^3+1}\Big|_0^1 = \frac{4\sqrt{2}-2}{9}$

h) $I_8 = \int\limits_0^1 (x^2-2x)\sqrt{x^3-3x^2+2}dx = \frac{1}{3}\int\limits_0^1 \sqrt{x^3-3x^2+2}d(x^3-3x^2+2)$

$= \frac{1}{3}.\frac{2}{3}(x^3-3x^2+2)\sqrt{x^3-3x^2+2}\Big|_0^1 = 0 - \frac{4\sqrt{2}}{9} = -\frac{4\sqrt{2}}{9}$

HT 3.Tính các tích phân sau:

a) $I_1 = \int\limits_1^4 \frac{dx}{\sqrt{x}}$ **b)** $I_2 = \int\limits_0^1 \frac{dx}{\sqrt{2x+1}}$ **c)** $I_3 = \int\limits_{-1}^0 \frac{dx}{\sqrt{1-2x}}$

d) $I_4 = \int\limits_0^1 \frac{(x+1)dx}{\sqrt{x^2+2x+2}}$ **e)** $I_5 = \int\limits_0^1 \frac{(x-2)dx}{\sqrt{x^2-4x+5}}$

Bài giải

a) $I_1 = \int\limits_1^4 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_1^4 = 4-2=2$

b) $I_2 = \int\limits_0^1 \frac{dx}{\sqrt{2x+1}} = \frac{1}{2}\int\limits_0^1 \frac{d(2x+1)}{\sqrt{2x+1}} = \sqrt{2x+1}\Big|_0^1 = \sqrt{3}-1$

c) $I_3 = \int\limits_{-1}^0 \frac{dx}{\sqrt{1-2x}} = -\frac{1}{2}\int\limits_{-1}^0 \frac{d(1-2x)}{\sqrt{1-2x}} = -\sqrt{1-2x}\Big|_{-1}^0 = -1+\sqrt{3}$

d) $I_4 = \int\limits_0^1 \frac{(x+1)dx}{\sqrt{x^2+2x+2}} = \frac{1}{2}\int\limits_0^1 \frac{d(x^2+2x+2)}{\sqrt{x^2+2x+2}} = \sqrt{x^2+2x+2}\Big|_0^1 = \sqrt{5}-\sqrt{2}$

e) $I_5 = \int\limits_0^1 \frac{(x-2)dx}{\sqrt{x^2-4x+5}} = \frac{1}{2}\int\limits_0^1 \frac{d(x^2-4x+5)}{\sqrt{x^2-4x+5}} = \sqrt{x^2-4x+5}\Big|_0^1 = \sqrt{2}-\sqrt{5}$

HT 4.Tính các tích phân sau:

a) $I_1 = \int\limits_1^e \frac{dx}{x}$ **b)** $I_2 = \int\limits_{-1}^0 \frac{dx}{1-2x}$ **c)** $I_3 = \int\limits_0^1 \frac{xdx}{x^2+1}$

d) $I_4 = \int\limits_0^1 \frac{(x+1)dx}{x^2+2x+2}$ **e)** $I_5 = \int\limits_0^1 \frac{x-2}{x^2-4x+5}dx$

Bài giải

a) $I_1 = \int\limits_1^e \frac{dx}{x} = \ln|x|\Big|_1^e = \ln e - \ln 1 = 1$

b) $I_2 = \int\limits_{-1}^0 \frac{dx}{1-2x} = -\frac{1}{2}\int\limits_{-1}^0 \frac{d(1-2x)}{1-2x} = -\frac{1}{2}\ln|1-2x|\Big|_{-1}^0 = -\frac{1}{2}(\ln 1 - \ln 3) = \frac{\ln 3}{2}$

c) $I_3 = \int\limits_0^1 \frac{xdx}{x^2+1} = \frac{1}{2}\int\limits_0^1 \frac{d\left(x^2+1\right)}{x^2+1} = \frac{1}{2}\ln\left|x^2+1\right|\Big|_0^1 = \frac{1}{2}(\ln 2 - \ln 1) = \frac{\ln 2}{2}$

d) $I_4 = \int\limits_0^1 \frac{(x+1)dx}{x^2+2x+2} = \frac{1}{2}\int\limits_0^1 \frac{d(x^2+2x+2)}{x^2+2x+2} = \frac{1}{2}\ln\left|x^2+2x+2\right|\Big|_0^1 = \frac{1}{2}(\ln 5 - \ln 2) = \frac{1}{2}\ln\frac{5}{2}$

e) $I_5 = \int\limits_0^1 \frac{x-2}{x^2-4x+5}dx = \frac{1}{2}\int\limits_0^1 \frac{d(x^2-4x+5)}{x^2-4x+5} = \frac{1}{2}\ln\left|x^2-4x+5\right|\Big|_0^1 = \frac{1}{2}(\ln 2 - \ln 5) = \frac{1}{2}\ln\frac{2}{5}$

HT 5.Tính các tích phân sau:

a) $I_1 = \int\limits_1^2 \frac{dx}{x^2}$ **b)** $I_2 = \int\limits_{-1}^0 \frac{dx}{(2x-1)^2}$ **c)** $I_3 = \int\limits_0^1 \frac{dx}{(3x+1)^2}$

Bài giải

a) $I_1 = \int\limits_1^2 \frac{dx}{x^2} = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

b) $I_2 = \int\limits_{-1}^0 \frac{dx}{(2x-1)^2} = \frac{1}{2}\int\limits_{-1}^0 \frac{d(2x-1)}{(2x-1)^2} = -\frac{1}{2}.\frac{1}{2x-1}\Big|_{-1}^0 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

c) $I_3 = \int\limits_0^1 \frac{dx}{(3x+1)^2} = \frac{1}{3}\int\limits_0^1 \frac{d(3x+1)}{(3x+1)^2} = -\frac{1}{3}.\frac{1}{3x+1}\Big|_0^1 = -\frac{1}{12} + \frac{1}{4} = \frac{1}{6}$

HT 6.Tính các tích phân sau:

a) $I_1 = \int\limits_0^1 e^{3x}dx$ **b)** $I_2 = \int\limits_0^1 e^x(2e^x+1)^3dx$ **c)** $I_3 = \int\limits_0^1 e^x(1-4e^x)^3dx$

d) $I_4 = \int\limits_0^1 \frac{e^xdx}{e^x+1}$ **e)** $I_5 = \int\limits_1^2 \frac{e^{2x}dx}{(e^{2x}-1)^2}$ **f)** $I_6 = \int\limits_1^2 \frac{e^{2x}dx}{(1-3e^{2x})^3}$

g) $I_7 = \int\limits_0^1 e^x\sqrt{2e^x+1}dx$ **h)** $I_8 = \int\limits_0^1 e^{2x}\sqrt{1+3e^{2x}}dx$ **i)** $I_9 = \int\limits_0^1 \frac{e^xdx}{\sqrt{e^x+1}}$

a) $I_1 = \int_0^1 e^{3x}dx = \frac{1}{3}e^{3x}\Big|_0^1 = \frac{e^3}{3} - \frac{1}{3}$

b) $I_2 = \int_0^1 e^x(2e^x+1)^3dx = \frac{1}{2}\int_0^1 (2e^x+1)^3 d(2e^x+1) = \frac{1}{2}.\frac{(2e^x+1)^4}{4}\Big|_0^1$

$= \frac{1}{2}\left(\frac{(2e+1)^4}{4} - \frac{81}{4}\right) = \frac{(2e+1)^4}{8} - \frac{81}{8}$

c) $I_3 = \int_0^1 e^x(1-4e^x)^3dx = -\frac{1}{4}\int_0^1 (1-4e^x)^3 d(1-4e^x)$

$= -\frac{1}{4}.\frac{(1-4e^x)^4}{4}\Big|_0^1 = -\frac{1}{4}\left(\frac{(1-4e)^4}{4} - \frac{81}{4}\right) = \frac{81-(1-4e)^4}{16}$

d) $I_4 = \int_0^1 \frac{e^xdx}{e^x+1} = \int_0^1 \frac{d(e^x+1)}{e^x+1} = \ln\left|e^x+1\right|\Big|_0^1 = \ln(e+1) - \ln 2 = \ln\frac{e+1}{2}$

e) $I_5 = \int_1^2 \frac{e^{2x}dx}{(e^{2x}-1)^2} = \frac{1}{2}\int_1^2 \frac{d(e^{2x}-1)}{(e^{2x}-1)^2} = -\frac{1}{2}.\frac{1}{e^{2x}-1}\Big|_1^2 = -\frac{1}{2(e^4-1)} + \frac{1}{2(e^2-1)} = \frac{e^2}{2(e^4-1)}$

f) $I_6 = \int_1^2 \frac{e^{2x}dx}{(1-3e^{2x})^3} = -\frac{1}{6}\int_1^2 \frac{d(1-3e^{2x})}{(1-3e^{2x})^3} = -\frac{1}{6}.\frac{-1}{2(1-3e^{2x})^2}\Big|_1^2 = \frac{1}{12(1-3e^4)} - \frac{1}{12(1-3e^2)}$

g) $I_7 = \int_0^1 e^x\sqrt{2e^x+1}dx = \frac{1}{2}\int_0^1 \sqrt{2e^x+1}d(2e^x+1) = \frac{1}{2}.\frac{2}{3}(2e^x+1)\sqrt{2e^x+1}\Big|_0^1 = \frac{1}{3}(2e+1)\sqrt{2e+1} - \sqrt{3}$

h) $I_8 = \int_0^1 e^{2x}\sqrt{1+3e^{2x}}dx = \frac{1}{6}\int_0^1 \sqrt{1+3e^{2x}}d(1+3e^{2x}) = \frac{1}{6}.\frac{2}{3}(1+3e^{2x})\sqrt{1+3e^{2x}}\Big|_0^1 = \frac{1}{9}(1+3e^2)\sqrt{1+3e^2} - \frac{8}{9}$

i) $I_9 = \int_0^1 \frac{e^xdx}{\sqrt{e^x+1}} = \int_0^1 \frac{d(e^x+1)}{\sqrt{e^x+1}} = 2\sqrt{e^x+1}\Big|_0^1 = 2\sqrt{e+1} - 2$

HT 7.Tính các tích phân sau:

a) $I_1 = \int_1^e \frac{\ln x}{x}dx$ **b)** $I_2 = \int_1^e \frac{3\ln x+1}{x}dx$ **c)** $I_3 = \int_1^e \frac{(3\ln x+1)^3}{x}dx$

d) $I_4 = \int_1^e \frac{4\ln^3x + 3\ln^2x - 2\ln x + 1}{x}dx$ **e)** $I_5 = \int_e^{e^2} \frac{dx}{x\ln x}$ **f)** $I_6 = \int_1^e \frac{dx}{x(3\ln x+1)}$

g) $I_7 = \int\limits_1^e \frac{\sqrt{3\ln x + 1}dx}{x}$ **h)** $I_8 = \int\limits_1^e \frac{dx}{x\sqrt{3\ln x + 1}}$

Bài giải

a) $I_1 = \int\limits_1^e \frac{\ln x}{x}dx = \int\limits_1^e \ln x d(\ln x) = \frac{\ln^2 x}{2}\Big|_1^e = \frac{\ln^2 e}{2} - \frac{\ln^2 1}{2} = \frac{1}{2}$

b) $I_2 = \int\limits_1^e \frac{3\ln x + 1}{x}dx = \int\limits_1^e (3\ln x + 1)d(\ln x) = \left(\frac{3\ln^2 x}{2} + \ln x\right)\Big|_1^e = (\frac{3}{2} + 1) - 0 = \frac{5}{2}$

c) $I_3 = \int\limits_1^e \frac{(3\ln x + 1)^3}{x}dx = \frac{1}{3}\int\limits_1^e (3\ln x + 1)^3 d(3\ln x + 1) = \frac{1}{3}.\frac{(3\ln x + 1)^4}{4}\Big|_1^e = \frac{64}{3} - \frac{1}{12} = \frac{85}{4}$

d) $I_4 = \int\limits_1^e \frac{4\ln^3 x + 3\ln^2 x - 2\ln x + 1}{x}dx = \int\limits_1^e (4\ln^3 x + 3\ln^2 x - 2\ln x + 1)d(\ln x)$

$= (\ln^4 x + \ln^3 x - \ln^2 x + \ln x)\Big|_1^e = (1 + 1 - 1 + 1) - 0 = 2$

e) $I_5 = \int\limits_e^{e^2} \frac{dx}{x\ln x} = \int\limits_e^{e^2} \frac{d(\ln x)}{\ln x} = \ln(\ln x)\Big|_e^{e^2} = \ln(\ln e^2) - \ln(\ln e) = \ln 2$

f) $I_6 = \int\limits_1^e \frac{dx}{x(3\ln x + 1)} = \frac{1}{3}\int\limits_1^e \frac{d(3\ln x + 1)}{3\ln x + 1} = \frac{1}{3}\ln(3\ln x + 1)\Big|_1^e = \frac{1}{3}(\ln 4 - \ln 1) = \frac{\ln 4}{3}$

g) $I_7 = \int\limits_1^e \frac{\sqrt{3\ln x + 1}dx}{x} = \frac{1}{3}\int\limits_1^e \sqrt{3\ln x + 1}d(3\ln x + 1) = \frac{1}{3}.\frac{2}{3}(3\ln x + 1)\sqrt{3\ln x + 1}\Big|_1^e = \frac{16}{9} - \frac{2}{9} = \frac{14}{9}$

h) $I_8 = \int\limits_1^e \frac{dx}{x\sqrt{3\ln x + 1}} = = \frac{1}{3}\int\limits_1^e \frac{d(3\ln x + 1)}{\sqrt{3\ln x + 1}} = \frac{1}{3}.2\sqrt{3\ln x + 1}\Big|_1^e = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

HT 8. Tính các tích phân sau:

a) $I_1 = \int\limits_0^{\frac{\pi}{2}} \cos^2 x \sin x dx$ **b)** $I_2 = \int\limits_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$ **c)** $I_3 = \int\limits_0^{\frac{\pi}{4}} \sin^3 2x \cos 2x dx$

d) $I_4 = \int\limits_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}dx$ **e)** $I_5 = \int\limits_0^{\frac{\pi}{2}} \sin x\sqrt{3\cos x + 1}dx$ **f)** $I_6 = \int\limits_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{3\sin x + 1}}dx$

Giải

a) $I_1 = \int\limits_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = -\int\limits_0^{\frac{\pi}{2}} \cos^2 x d(\cos x) = -\frac{\cos^3 x}{3}\Bigg|_0^{\frac{\pi}{2}} = \frac{1}{3}$

b) $I_2 = \int\limits_0^{\frac{\pi}{2}} \sin^2 x \cos x dx = \int\limits_0^{\frac{\pi}{2}} \sin^2 x d(\sin x) = \frac{\sin^3 x}{3}\Bigg|_0^{\frac{\pi}{2}} = \frac{1}{3}$

c) $I_3 = \int\limits_0^{\frac{\pi}{4}} \sin^3 2x \cos 2x dx = \frac{1}{2}\int\limits_0^{\frac{\pi}{4}} \sin^3 2x d(\sin 2x) = \frac{\sin^4 2x}{8}\Bigg|_0^{\frac{\pi}{4}} = \frac{1}{8}$

d) $I_4 = \int\limits_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx = -\int\limits_0^{\frac{\pi}{4}} \frac{d(\cos x)}{\cos x} = -\ln(\cos x)\Bigg|_0^{\frac{\pi}{4}} = -\ln\frac{\sqrt{2}}{2} + \ln 1 = -\ln\frac{\sqrt{2}}{2}$

e) $I_5 = \int\limits_0^{\frac{\pi}{2}} \sin x\sqrt{3\cos x + 1}dx = \frac{1}{3}\int\limits_0^{\frac{\pi}{2}} \sqrt{3\cos x + 1}d(3\cos x + 1) = \frac{1}{2}.\frac{2}{3}(3\cos x + 1)\sqrt{3\cos x + 1}\Bigg|_0^{\frac{\pi}{2}} = \frac{1}{3} - \frac{4}{3} = -1$

f) $I_6 = \int\limits_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{3\sin x + 1}} dx = \frac{1}{3}\int\limits_0^{\frac{\pi}{2}} \frac{d(3\sin x + 1)}{\sqrt{3\sin x + 1}} = \frac{2}{3}\sqrt{3\sin x + 1}\Bigg|_0^{\frac{\pi}{2}} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

PHẦN II TÍCH PHẦN HÀM HỮU TỶ

http://www.Luuhuythuong.blogspot.com

I.DẠNG 1: $\int \frac{dx}{ax+b} = \frac{1}{a}\ln\left|ax+b\right| + C$

HT 1.Tính các tích phân sau:

a) $\int\limits_0^1 \frac{dx}{3x+1}$ b) $\int\limits_{-1}^0 \frac{dx}{1-3x}$ c) $\int\limits_0^1 \left(\frac{1}{2x+1} - \frac{3}{4-2x}\right)dx$

Giải

a) $\int\limits_0^1 \frac{dx}{3x+1} = \frac{1}{3}\ln\left|3x+1\right|\Big|_0^1 = \frac{1}{3}(\ln 4 - \ln 1) = \frac{\ln 4}{3}$

b) $\int\limits_{-1}^0 \frac{dx}{1-3x} = -\frac{1}{3}\ln\left|1-3x\right|\Big|_{-1}^0 = -\frac{1}{3}(\ln 1 - \ln 4) = -\frac{\ln 4}{3}$

c) $\int\limits_0^1 \left(\frac{1}{2x+1} - \frac{3}{4-2x}\right)dx = \left(\frac{1}{2}\ln\left|2x+1\right| + \frac{3}{2}\ln\left|4-2x\right|\right)\Big|_0^1 = \left(\frac{1}{2}\ln 3 + \frac{3}{2}\ln 2\right) - \left(\frac{1}{2}\ln 1 + \frac{3}{2}\ln 4\right)$

$= \frac{1}{2}\ln 3 + \frac{3}{2}\ln\frac{1}{2}$

HT 2.Tính các tích phân sau:

a) $I_1 = \int\limits_1^2 \frac{x^4 + 3x^3 - 2x^2 + 5x - 1}{x^2}dx$ **b)** $I_2 = \int\limits_0^1 \frac{x^3 - 3x^2 + 2x - 1}{x-2}dx$ **c)** $I_3 = \int\limits_{-1}^0 \frac{2x^3 - 3x^2 + 4x - 1}{1-2x}$

Giải

a) $I_1 = \int\limits_1^2 \frac{x^4 + 3x^3 - 2x^2 + 5x - 1}{x^2}dx = \int\limits_1^2 (x^2 + 3x - 2 + \frac{5}{x} - \frac{1}{x^2})dx$

$= \left(\frac{x^3}{3} + \frac{3x^2}{2} - 2x + 5\ln x + \frac{1}{x}\right)\Big|_1^2 = \left(\frac{8}{3} + 6 - 4 + 5\ln 2 + \frac{1}{2}\right) - \left(\frac{1}{3} + \frac{3}{2} - 2 + 5\ln 1 + 1\right) = \frac{13}{3} + 5\ln 2$

b) $I_2 = \int\limits_0^1 \frac{x^3 - 3x^2 + 2x - 1}{x-2}dx = \int\limits_0^1 \left(x^2 - x - \frac{1}{x-2)}\right)dx$

$= \left(\frac{x^3}{3} - \frac{x^2}{2} - \ln\left|x-2\right|\right)\Big|_0^1 = \left(\frac{1}{3} - \frac{1}{2} - \ln 1\right) - \left(-\ln 2\right) = \ln 2 - \frac{1}{6}$

c) $I_3 = \int\limits_{-1}^0 \frac{2x^3 - 3x^2 + 4x - 1}{1-2x} = \int\limits_{-1}^0 \left(-x^2 + x - \frac{3}{2} + \frac{1}{2(-2x+1)}\right)dx$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}-\frac{3}{2}x-\frac{1}{4}\ln\left|-2x+1\right|\right)\Bigg|_{-1}^{0}$$

$$=(-\frac{1}{4}\ln 1)-(\frac{1}{3}+\frac{1}{2}+\frac{3}{2}-\frac{1}{4}\ln 3)=\frac{\ln 3}{4}-\frac{7}{3}$$

II.DẠNG 2: $\int \frac{dx}{ax^2+bx+c}$

HT 3.Tính các tích phân sau (mẫu số có hai nghiệm phân biệt)

a) $\int_0^1 \frac{dx}{(x+1)(x+2)}$ **b)** $\int_0^1 \frac{dx}{(x+1)(3-x)}$ **c)** $\int_0^1 \frac{dx}{(x+1)(2x+3)}$

Giải

a) $\int_0^1 \frac{dx}{(x+1)(x+2)}=\int_0^1 \frac{(x+2)-(x+1)}{(x+1)(x+2)}dx=\int_0^1\left(\frac{1}{x+1}-\frac{1}{x+2}\right)dx$

$$=\left(\ln\left|x+1\right|-\ln\left|x+2\right|\right)\Big|_0^1=\ln\left|\frac{x+1}{x+2}\right|\Bigg|_0^1=\ln\frac{2}{3}-\ln\frac{1}{2}=\ln\frac{4}{3}$$

b) $\int_0^1 \frac{dx}{(x+1)(3-x)}=\frac{1}{4}\int_0^1 \frac{(x+1)+(3-x)}{(x+1)(3-x)}dx=\frac{1}{4}\int_0^1\left(\frac{1}{3-x}+\frac{1}{x+1}\right)dx$

$$=\frac{1}{4}\left(-\ln\left|3-x\right|+\ln\left|x+1\right|\right)\Big|_0^1=\frac{1}{4}\ln\left|\frac{x+1}{3-x}\right|\Bigg|_0^1=\frac{1}{4}\left(\ln 1-\ln\frac{1}{3}\right)=-\frac{\ln 3}{4}$$

c) $\int_0^1 \frac{dx}{(x+1)(2x+3)}=\int_0^1 \frac{(2x+3)-2(x+1)}{(x+1)(2x+3)}dx=\int_0^1\left(\frac{1}{x+1}-\frac{2}{2x+3}\right)dx$

$$=\left(\ln\left|x+1\right|-\ln\left|2x+3\right|\right)\Big|_0^1=\ln\left|\frac{x+1}{2x+3}\right|\Bigg|_0^1=\ln\frac{2}{5}-\ln\frac{1}{3}=\ln\frac{6}{5}$$

HT 4.Tính các tích phân sau:

a) $\int_0^1 \frac{dx}{x^2-x-12}$ b) $\int_{-1}^0 \frac{dx}{2x^2-5x+2}$ c) $\int_1^2 \frac{dx}{1-2x-3x^2}$

Giải

a) $\int_0^1 \frac{dx}{x^2-x-12}$ **=** $\int_0^1 \frac{dx}{(x+3)(x-4)}=\frac{1}{7}\int_0^1 \frac{(x+3)-(x-4)}{(x+3)(x-4)}dx$

$$=\frac{1}{7}\int_0^1\left(\frac{1}{x-4}-\frac{1}{x+3}\right)dx=\frac{1}{7}\left(\ln\left|x-4\right|-\ln\left|x+3\right|\right)\Big|_0^1=\frac{1}{7}\ln\left|\frac{x-4}{x+3}\right|\Bigg|_0^1$$

$$=\frac{1}{7}(\ln\frac{3}{4}-\ln\frac{4}{3})=\frac{1}{7}\ln\frac{9}{16}$$

b) $\int_{-1}^0 \frac{dx}{2x^2-5x+2}$ **=** $\int_{-1}^0 \frac{dx}{2(x-2)(x-\frac{1}{2})}=\int_{-1}^0 \frac{dx}{(x-2)(2x-1)}=\frac{1}{3}\int_{-1}^0 \frac{(2x-1)-2(x-2)}{(x-2)(2x-1)}dx$

$$= \frac{1}{3}\int_{-1}^{0}\left(\frac{1}{x-2}-\frac{2}{2x-1}\right)dx = \frac{1}{3}\left(\ln|x-2|-\ln|2x-1|\right)\Big|_{-1}^{0}$$

$$= \frac{1}{3}\ln\left|\frac{x-2}{2x-1}\right|\Big|_{-1}^{0} = \frac{1}{3}(\ln 2-\ln 1) = \frac{\ln 2}{3}$$

c) $\displaystyle\int_1^2 \frac{dx}{1-2x-3x^2} = \int_1^2 \frac{dx}{-3(x+1)(x-\frac{1}{3})} = \int_1^2 \frac{dx}{(x+1)(1-3x)} = \frac{1}{4}\int_1^2 \frac{3(x+1)+(1-3x)}{(x+1)(1-3x)}dx$

$$= \frac{1}{4}\int_1^2\left(\frac{3}{1-3x}+\frac{1}{x+1}\right)dx = \frac{1}{4}\left(-\ln|1-3x|+\ln|x+1|\right)\Big|_1^2 = \frac{1}{4}\ln\left|\frac{x+1}{1-3x}\right|\Big|_1^2 = \frac{1}{4}(\ln\frac{3}{5}-\ln 1) = \frac{1}{4}\ln\frac{3}{5}$$

HT 5. Tính các tích phân sau: (Mẫu số có nghiệm kép)

a) $\displaystyle\int_1^2 \frac{dx}{x^2}$ b) $\displaystyle\int_0^1 \frac{dx}{(3x+1)^2}$ c) $\displaystyle\int_{-1}^0 \frac{dx}{(1-2x)^2}$ d) $\displaystyle\int_{-1}^0 \frac{dx}{9x^2-6x+1}$ e) $\displaystyle\int_{-1}^0 \frac{dx}{-16x^2+8x-1}$

Giải

a) $\displaystyle\int_1^2 \frac{dx}{x^2} = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2}+1 = \frac{1}{2}$

b) $\displaystyle\int_0^1 \frac{dx}{(3x+1)^2} = -\frac{1}{3}.\frac{1}{(3x+1)}\Big|_0^1 = -\left(\frac{1}{12}-\frac{1}{3}\right) = \frac{1}{4}$

c) $\displaystyle\int_{-1}^0 \frac{dx}{(1-2x)^2} = \int_{-1}^0 \frac{dx}{(2x-1)^2} = -\frac{1}{2}.\frac{1}{2x-1}\Big|_{-1}^0 = -\left(-\frac{1}{2}+\frac{1}{6}\right) = \frac{1}{3}$

d) $\displaystyle\int_{-1}^0 \frac{dx}{9x^2-6x+1} = \int_{-1}^0 \frac{dx}{(3x-1)^2} = -\frac{1}{3}.\frac{1}{3x-1}\Big|_{-1}^0 = -\left(-\frac{1}{3}+\frac{1}{12}\right) = \frac{1}{4}$

e) $\displaystyle\int_{-1}^0 \frac{dx}{-16x^2+8x-1} = -\int_{-1}^0 \frac{dx}{16x^2-8x+1} = -\int_{-1}^0 \frac{dx}{(4x-1)^2} = \frac{1}{4}.\frac{1}{4x-1}\Big|_{-1}^0 = -\frac{1}{4}+\frac{1}{20} = -\frac{1}{5}$

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HT 6. Tính các tích phân sau: (Mẫu số vô nghiệm)

a) $\displaystyle I_1 = \int_0^1 \frac{dx}{x^2+1}$ b) $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{x^2+3}$ c) $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{2x^2+3}$

Giải

a) $\displaystyle I_1 = \int_0^1 \frac{dx}{x^2+1}$

Đặt: $x = \tan t \quad \left(t \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right)\right)$

$\Rightarrow dx = \dfrac{dt}{\cos^2 t}$

Đổi cận: Với $x = 0 \Rightarrow t = 0$

Với $x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t(\tan^2 t + 1)} = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t.\frac{1}{\cos^2 t}} = \int_0^{\frac{\pi}{4}} dt = t\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

b) $I_2 = \int_0^{\sqrt{3}} \frac{dx}{x^2 + 3}$

Đặt: $x = \sqrt{3}\tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\Rightarrow dx = \frac{\sqrt{3}dt}{\cos^2 t}$$

Đổi cận: Với $x = 0 \Rightarrow t = 0$; Với $x = \sqrt{3} \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow I_2 = \int_0^{\frac{\pi}{4}} \frac{\sqrt{3}dt}{\cos^2 t(3\tan^2 t + 3)} = \frac{\sqrt{3}}{3}\int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t.\frac{1}{\cos^2 t}} = \frac{\sqrt{3}}{3}\int_0^{\frac{\pi}{4}} dt = \frac{\sqrt{3}}{3}t\Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{3}\pi}{12}$$

c) $I_3 = \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{2x^2 + 3} = \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{2\left(x^2 + \frac{3}{2}\right)} = \frac{1}{2}\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{x^2 + \frac{3}{2}}$

Đặt: $x = \sqrt{\frac{3}{2}}\tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\Rightarrow dx = \frac{\sqrt{6}}{2}\frac{dt}{\cos^2 t}$$

Đổi cận: Với $x = 0 \Rightarrow t = 0$; Với $x = \frac{\sqrt{2}}{2} \Rightarrow t = \frac{\pi}{6}$

$$\Rightarrow I_3 = \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{\sqrt{6}dt}{2\cos^2 t(\frac{3}{2}\tan^2 t + \frac{3}{2})} = \frac{\sqrt{6}}{6}\int_0^{\frac{\pi}{6}} \frac{dt}{\cos^2 t.\frac{1}{\cos^2 t}} = \frac{\sqrt{6}}{6}\int_0^{\frac{\pi}{6}} dt = \frac{\sqrt{6}}{6}t\Big|_0^{\frac{\pi}{6}} = \frac{\sqrt{6}\pi}{36}$$

HT 7. Tính các tích phân sau: (Mẫu số vô nghiệm)

a) $I_1 = \int_{-1}^{0} \frac{dx}{(x+1)^2 + 1}$ b) $I_2 = \int_2^4 \frac{dx}{x^2 - 4x + 8}$ c) $I_3 = \int_0^1 \frac{dx}{x^2 + x + 1}$

Giải

a) $I_1 = \int_{-1}^{0} \frac{dx}{(x+1)^2 + 1}$

Đặt: $x + 1 = \tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$\Rightarrow dx = \dfrac{dt}{\cos^2 t}$

Đổi cận: Với $x = -1 \Rightarrow t = 0$; Với $x = 0 \Rightarrow t = \dfrac{\pi}{4}$

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t(\tan^2 t + 1)} = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t.\dfrac{1}{\cos^2 t}} = \int_0^{\frac{\pi}{4}} dt = t\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

b) $I_2 = \displaystyle\int_2^4 \frac{dx}{x^2 - 4x + 8} = \int_2^4 \frac{dx}{(x-2)^2 + 4}$

Đặt: $x - 2 = 2\tan t$ Với $t \in \left(-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right)$

$\Rightarrow dx = \dfrac{2dt}{\cos^2 t}$

Đổi cận: Với $x = 2 \Rightarrow t = 0$; Với $x = 4 \Rightarrow t = \dfrac{\pi}{4}$

$$\Rightarrow I_2 = \int_0^{\frac{\pi}{4}} \frac{2dt}{\cos^2 t(4\tan^2 t + 4)} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t.\dfrac{1}{\cos^2 t}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} dt = \frac{1}{2}t\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{8}$$

c) $I_3 = \displaystyle\int_0^1 \frac{dx}{x^2 + x + 1} = \int_0^1 \frac{dx}{\left(x + \dfrac{1}{2}\right)^2 + \dfrac{3}{4}}$

Đặt: $x + \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\tan t$ Với $t \in \left(-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right)$

$\Rightarrow dx = \dfrac{\sqrt{3}}{2}.\dfrac{dt}{\cos^2 t}$

Đổi cận: Với $x = 0 \Rightarrow t = \dfrac{\pi}{6}$;Với $x = 1 \Rightarrow t = \dfrac{\pi}{3}$

$$\Rightarrow I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{3}dt}{2\cos^2 t(\frac{3}{4}\tan^2 t + \frac{3}{4})} = \frac{2\sqrt{3}}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dt}{\cos^2 t.\dfrac{1}{\cos^2 t}} =$$

$$\frac{2\sqrt{3}}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dt = \frac{2\sqrt{3}}{3}t\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{2\sqrt{3}\pi}{9} - \frac{2\sqrt{3}\pi}{18} = \frac{2\sqrt{3}\pi}{18}$$

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III.Dạng 3: $\int \frac{mx+n}{ax^2+bx+c}dx$

HT 8.Tính các tích phân sau: (Mẫu số có 2 nghiệm phân biệt)

a) $I_1 = \int_0^1 \frac{x-1}{x^2+4x+3}dx$ b) $I_2 = \int_{-1}^0 \frac{2x+10}{-x^2+x+2}dx$ c) $I_3 = \int_{-1}^0 \frac{7-4x}{-2x^2-3x+2}dx$

Giải

a) $I_1 = \int_0^1 \frac{x-1}{x^2+4x+3}dx = \int_0^1 \frac{(x-1)dx}{(x+1)(x+3)}$

Xét đồng nhất thức: $\frac{x-1}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} = \frac{Ax+A+Bx+3B}{(x+3)(x+1)} = \frac{(A+b)x+A+3B}{(x+3)(x+1)}$

Đồng nhất thức hai vế ta được: $\begin{cases} A+B=1 \\ A+3B=-1 \end{cases} \Leftrightarrow \begin{cases} A=2 \\ B=-1 \end{cases}$

Vậy, $I_1 = \int_0^1 \left(\frac{2}{x+3} - \frac{1}{x+1}\right)dx = \left(2\ln\left|x+3\right| - \ln\left|x+1\right|\right)\Big|_0^1$

$= (2\ln 4 - \ln 2) - (2\ln 3 - \ln 1) = 2\ln\frac{4}{3} - \ln 2$

b) $\int_{-1}^0 \frac{2x+10}{-x^2+x+2}dx = \int_{-1}^0 \frac{2x+10}{(x+2)(1-x)}dx$

Xét đồng nhất thức: $\frac{2x+10}{(x+2)(1-x)} = \frac{A}{x+2} + \frac{B}{1-x} = \frac{A-Ax+Bx+2B}{(x+2)(1-x)} = \frac{(B-A)x+A+2B}{(x+2)(1-x)}$

Đồng nhất thức hai vế ta được: $\begin{cases} B-A=2 \\ A+2B=10 \end{cases} \Leftrightarrow \begin{cases} A=2 \\ B=4 \end{cases}$

Vậy, $I_2 = \int_{-1}^0 \left(\frac{2}{x+2} + \frac{4}{1-x}\right)dx = \left(2\ln\left|x+2\right| - 4\ln\left|1-x\right|\right)\Big|_{-1}^0$

$= (2\ln 2 - 4\ln 1) - (2\ln 1 - 4\ln 2) = 2\ln 2 + 4\ln 2 = \ln 4 + \ln 16 = \ln 64$

c) $I_3 = \int_{-1}^0 \frac{7-4x}{-2x^2-3x+2}dx = \int_{-1}^0 \frac{7-4x}{(x+2)(1-2x)}dx$

Xét đồng nhất thức: $\frac{7-4x}{(x+2)(1-2x)} = \frac{A}{x+2} + \frac{B}{1-2x} = \frac{A-2Ax+Bx+2B}{(x+2)(1-2x)} = \frac{(B-2A)x+A+2B}{(x+2)(1-2x)}$

Đồng nhất thức hai vế ta được: $\begin{cases} B-2A=-4 \\ A+2B=7 \end{cases} \Leftrightarrow \begin{cases} A=3 \\ B=2 \end{cases}$

Vậy, $I_3 = \int_{-1}^0 \left(\frac{2}{1-2x} + \frac{3}{x+2}\right)dx = \left(-\ln\left|1-2x\right| + 3\ln\left|x+2\right|\right)\Big|_{-1}^0$

$= (-\ln 1 + 2\ln 2) - (-\ln 3 + 3\ln 2) = \ln 3 - \ln 2 = \ln\frac{3}{2}$

HT 9. Tính các tích phân sau: (Mẫu số có nghiệm kép)

a) $I_1 = \int\limits_0^1 \frac{(3x+1)dx}{x^2+2x+1}$ b) $I_2 = \int\limits_{-1}^0 \frac{3x-1}{4x^2-4x+1}dx$ c) $I_3 = \int\limits_0^1 \frac{3x+2}{4x^2+12x+9}dx$

Giải

a) $I_1 = \int\limits_0^1 \frac{(3x+1)dx}{x^2+2x+1} = \int\limits_0^1 \frac{3x+1}{(x+1)^2}dx = \int\limits_0^1 \frac{3(x+1)-2}{(x+1)^2}dx = \int\limits_0^1 \left(\frac{3}{x+1} - \frac{2}{(x+1)^2}\right)dx$

$= \left(3\ln\left|x+1\right| + \frac{2}{x+1}\right)\Big|_0^1 = (3\ln 2 + 1) - (3\ln 1 + 2) = 3\ln 2 - 1$

b) $I_2 = \int\limits_{-1}^0 \frac{3x-1}{4x^2-4x+1}dx = \int\limits_{-1}^0 \frac{3x-1}{(2x-1)^2}dx = \int\limits_{-1}^0 \frac{\frac{3}{2}(2x-1)+\frac{1}{2}}{(2x-1)^2}dx$

$= \int\limits_{-1}^0 \left(\frac{3}{2}.\frac{1}{2x-1} + \frac{1}{2}.\frac{1}{(2x-1)^2}\right)dx = \left(\frac{3}{4}\ln\left|2x-1\right| - \frac{1}{4}.\frac{1}{2x-1}\right)\Big|_{-1}^0$

$= \left(\frac{3}{4}\ln 1 + \frac{1}{4}\right) - \left(\frac{3}{4}\ln 3 + \frac{1}{12}\right) = -\frac{3}{4}\ln 3 + \frac{1}{6}$

c) $I_3 = \int\limits_0^1 \frac{3x+2}{4x^2+12x+9}dx = \int\limits_0^1 \frac{3x+2}{(2x+3)^2}dx = \int\limits_0^1 \frac{\frac{3}{2}(2x+3)-\frac{5}{2}}{(2x+3)^2}dx$

$= \int\limits_0^1 \left(\frac{3}{2}.\frac{1}{2x+3} - \frac{5}{2}.\frac{1}{(2x+3)^2}\right)dx = \left(\frac{3}{4}\ln\left|2x+3\right| + \frac{5}{4}.\frac{1}{2x+3}\right)\Big|_0^1$

$= \left(\frac{3}{4}\ln 5 + \frac{1}{4}\right) - \left(\frac{3}{4}\ln 3 + \frac{5}{12}\right) = \frac{3}{4}\ln\frac{5}{3} - \frac{1}{6}$

HT 10. Tính các tích phân sau: (Mẫu số vô nghiệm)

a) $I_1 = \int\limits_0^1 \frac{3x+1}{x^2+1}dx$ b) $I_2 = \int\limits_1^3 \frac{3x+2}{x^2-4x+5}dx$ c) $I_3 = \int\limits_0^1 \frac{3x-1}{4x^2-4x+2}dx$

Giải

a) $I_1 = \int\limits_0^1 \frac{3x+1}{x^2+1}dx$

Chú ý: $(x^2+1)' = 2x$ Nên: $I_1 = \int\limits_0^1 \frac{\frac{3}{2}.2x+1}{x^2+1}dx = \int\limits_0^1 \left(\frac{3}{2}.\frac{2x}{x^2+1} + \frac{1}{x^2+1}\right)dx = \frac{3}{2}\int\limits_0^1 \frac{2x}{x^2+1}dx + \int\limits_0^1 \frac{dx}{x^2+1}$

Xét: $M = \frac{3}{2}\int\limits_0^1 \frac{2x}{x^2+1}dx = \frac{3}{2}\int\limits_0^1 \frac{d(x^2+1)}{x^2+1} = \frac{3}{2}\ln\left|x^2+1\right|\Big|_0^1 = \frac{3}{2}(\ln 2 - \ln 1) = \frac{3\ln 2}{2}$

Xét: $N = \int\limits_0^1 \frac{dx}{x^2+1}$

Đặt: $x = \tan t \quad \left(t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)\right)$

$\Rightarrow dx = \frac{dt}{\cos^2 t}$

Đổi cận: Với $x = 0 \Rightarrow t = 0$

Với $x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow M = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t(\tan^2 t + 1)} = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t.\frac{1}{\cos^2 t}} = \int_0^{\frac{\pi}{4}} dt = t\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

Vậy, $I_1 = M + N = \frac{3\ln 2}{2} + \frac{\pi}{4}$

b) $I_2 = \int_1^3 \frac{3x+2}{x^2 - 4x + 5}dx$

Chú ý: $(x^2 - 4x + 5)' = 2x - 4$

Khi đó: $I_2 = \int_1^3 \frac{\frac{3}{2}(2x-4)+8}{x^2-4x+5}dx = \int_1^3 \left(\frac{3}{2}\frac{2x-4}{x^2-4x+5} + 8.\frac{1}{x^2-4x+5}\right)dx$

$$= \frac{3}{2}\int_1^3 \frac{2x-4}{x^2-4x+5}dx + 8\int_1^3 \frac{1}{x^2-4x+5}dx$$

+ Xét: $M = \frac{3}{2}\int_1^3 \frac{2x-4}{x^2-4x+5}dx = \frac{3}{2}\int_1^3 \frac{d(x^2-4x+5)}{x^2-4x+5} = \frac{3}{2}\ln\left|x^2-4x+5\right|\Big|_1^3 = \frac{3}{2}(\ln 2 - \ln 2) = 0$

+ Xét: $N = 8\int_1^3 \frac{1}{x^2-4x+5}dx = 8\int_1^3 \frac{dx}{(x-2)^2+1}$

Đặt: $x - 2 = \tan t$ Với $\left(t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)\right)$

$\Rightarrow dx = \frac{dt}{\cos^2 t}$

Đổi cận: Với $x = 1 \Rightarrow t = -\frac{\pi}{4}$; Với $x = 3 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow N = 8\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dt}{\cos^2 t(\tan^2 t + 1)} = 8\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dt = 8t\Big|_{\frac{-\pi}{4}}^{\frac{\pi}{4}} = 4\pi$$

Vậy, $I_2 = M + N = 4\pi$

c) $I_3 = \int\limits_0^1 \frac{3x-1}{4x^2-4x+2}dx$

Chú ý: $(4x^2-4x+2)' = 8x-4$

Ta có: $I_3 = \int\limits_0^1 \frac{3x-1}{4x^2-4x+2}dx = \int\limits_0^1 \frac{\frac{3}{8}(8x-4)+\frac{1}{2}}{4x^2-4x+2}dx$

$= \frac{3}{8}\int\limits_0^1 \frac{8x-4}{4x^2-4x+2}dx + \frac{1}{2}\int\limits_0^1 \frac{dx}{4x^2-4x+2}$

+) Xét: $M = \frac{3}{8}\int\limits_0^1 \frac{8x-4}{4x^2-4x+2}dx = \frac{3}{8}\int\limits_0^1 \frac{d(4x^2-4x+2)}{4x^2-4x+2} = \frac{3}{8}\ln\left|4x^2-4x+2\right|\Big|_0^1 = \frac{3}{8}(\ln 2 - \ln 2) = 0$

+) Xét: $N = \frac{1}{2}\int\limits_0^1 \frac{dx}{4x^2-4x+2} = \frac{1}{2}\int\limits_0^1 \frac{dx}{(2x-1)^2+1}$

Đặt: $2x-1 = \tan t$ Với $\left(t \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right)\right)$

$\Rightarrow 2dx = \frac{dt}{\cos^2 t} \Leftrightarrow dx = \frac{dt}{2\cos^2 t}$

Đổi cận:Với $x = 0 \Rightarrow t = -\frac{\pi}{4}$;Với $x = 1 \Rightarrow t = \frac{\pi}{4}$

$\Rightarrow N = \frac{1}{2}\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dt}{2\cos^2 t(\tan^2 t+1)} = \frac{1}{2}\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dt = \frac{1}{2}t\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4}$

Vậy, $I_3 = M + N = \frac{\pi}{4}$

HT 11.Tính các tích phân sau:

a) $I_1 = \int\limits_{-1}^0 \frac{x^3-5x^2+6x-1}{x^2-3x+2}dx$ b) $I_2 = \int\limits_0^1 \frac{x^4+5x^3-3x^2+2x-1}{x^2+2x+1}dx$

c) $I_3 = \int\limits_{-1}^0 \frac{x^3+3x^2-6x+1}{x^2+2x+2}dx$ d) $I = \int\limits_1^2 \frac{x^2}{x^2-7x+12}dx$

Giải

a) $I_1 = \int_{-1}^{0} \frac{x^3 - 5x^2 + 6x - 1}{x^2 - 3x + 2} dx = \int_{-1}^{0} \left(x - 2 + \frac{-2x + 3}{x^2 - 3x + 2} \right) dx = \int_{-1}^{0} (x - 2) dx + \int_{-1}^{0} \frac{-2x + 3}{x^2 - 3x + 2} dx$

+) Xét: $M = \int_{-1}^{0} (x - 2) dx = \left(\frac{x^2}{2} - 2x \right) \Big|_{-1}^{0} = -\left(\frac{1}{2} + 2 \right) = -\frac{5}{2}$

+) Xét: $N = \int_{-1}^{0} \frac{-2x + 3}{x^2 - 3x + 2} dx = \int_{-1}^{0} \frac{-2x + 3}{(x - 1)(x - 2)} dx$

Dùng đồng nhất thức ta tách được:

$N = \int_{-}^{0} \left(\frac{-1}{x - 1} + \frac{-1}{x - 2} \right) dx = \left(-\ln|x - 1| - \ln|x - 2| \right) \Big|_{-1}^{0} = (-\ln 1 - \ln 2) - (-\ln 2 - \ln 3) = \ln 3$

Vậy, $I_1 = M + N = \ln 3 - \frac{5}{2}$

b) $I_2 = \int_{0}^{1} \frac{x^4 + 5x^3 - 3x^2 + 2x - 1}{x^2 + 2x + 1} dx = \int_{0}^{1} (x^2 + 3x - 10 + \frac{19x + 9}{x^2 + 2x + 1}) dx$

+) Xét: $M = \int_{0}^{1} (x^2 + 3x - 10) dx = \left(\frac{x^3}{3} + \frac{3x^2}{2} - 10x \right) \Big|_{0}^{1} = (\frac{1}{3} + \frac{3}{2} - 10) - 0 = -\frac{49}{6}$

+) Xét: $N = \int_{0}^{1} \frac{19x + 9}{x^2 + 2x + 1} dx = \int_{0}^{1} \frac{19(x + 1) - 10}{(x + 1)^2} dx = \int_{0}^{1} \left(\frac{19}{x + 1} - \frac{10}{(x + 1)^2} \right) dx$

$= \left(19\ln|x + 1| + \frac{10}{x + 1} \right) \Big|_{0}^{1} = (19\ln 2 + 5) - (19\ln 1 + 10) = 19\ln 2 - 5$

Vậy, $I_2 = M + N = 19\ln 2 - \frac{79}{6}$

c) $I_3 = \int_{-1}^{0} \frac{x^3 + 3x^2 - 6x + 1}{x^2 + 2x + 2} dx = \int_{-1}^{0} \left(x + 1 - \frac{10x + 1}{x^2 + 2x + 2} \right) dx$

+) Xét: $M = \int_{-1}^{0} (x + 1) dx = \left(\frac{x^2}{2} + x \right) \Big|_{-1}^{0} = -\left(\frac{1}{2} - 1 \right) = \frac{1}{2}$

+) Xét: $N = \int_{-1}^{0} \frac{10x + 1}{x^2 + 2x + 2} dx = \int_{-1}^{0} \frac{5(2x + 2) - 9}{x^2 + 2x + 2} dx = \int_{-1}^{0} \left(\frac{5(2x + 2)}{x^2 + 2x + 2} - \frac{9}{x^2 + 2x + 2} \right) dx$

$P = 5\int_{-1}^{0} \frac{2x + 2}{x^2 + 2x + 2} dx = 5\int_{-1}^{0} \frac{d(x^2 + 2x + 2)}{x^2 + 2x + 2} = 5\ln|x^2 + 2x + 2| \Big|_{-1}^{0} = 5(\ln 2 - \ln 1) = 5\ln 2$

$$Q = 9\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2} = 9\int_{-1}^{0} \frac{dx}{(x+1)^2 + 1}$$

Đặt: $x + 1 = \tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\Rightarrow dx = \frac{dt}{\cos^2 t}$$

Đổi cận: Với $x = -1 \Rightarrow t = 0$; Với $x = 0 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow Q = 9\int_{0}^{\frac{\pi}{4}} \frac{dt}{\cos^2 t(\tan^2 t + 1)} = 9\int_{0}^{\frac{\pi}{4}} dt = 9t\Big|_0^{\frac{\pi}{4}} = \frac{9\pi}{4}$$

$$\Rightarrow N = P - Q = 5\ln 2 - \frac{9\pi}{4} \Rightarrow I_3 = M + N = \frac{1}{2} + 5\ln 2 - \frac{9\pi}{4}$$

d) $I = \int_{1}^{2} \left(1 + \frac{16}{x-4} - \frac{9}{x-3}\right)dx = \left(x + 16\ln|x-4| - 9\ln|x-3|\right)\Big|_1^2 = 1 + 25\ln 2 - 16\ln 3$.

HT 12.Tính các tích phân sau:

a) $I = \int_{1}^{2} \frac{dx}{x^5 + x^3}$ *b)* $I = \int_0^1 \frac{xdx}{(x+1)^3}$

Giải

a) $I = \int_{1}^{2} \frac{dx}{x^5 + x^3}$

Ta có: $\frac{1}{x^3(x^2+1)} = -\frac{1}{x} + \frac{1}{x^3} + \frac{x}{x^2+1}$

$$\Rightarrow I = \left[-\ln|x| - \frac{1}{2x^2} + \frac{1}{2}\ln(x^2+1)\right]\Bigg|_1^2 = -\frac{3}{2}\ln 2 + \frac{1}{2}\ln 5 + \frac{3}{8}$$

b) $I = \int_0^1 \frac{xdx}{(x+1)^3}$

Ta có: $\frac{x}{(x+1)^3} = \frac{x+1-1}{(x+1)^3} = (x+1)^{-2} - (x+1)^{-3}$

$$\Rightarrow I = \int_0^1 \left[(x+1)^{-2} - (x+1)^{-3}\right]dx = \frac{1}{8}$$

HT 13.Tính các tích phân sau: (Đổi biến số)

1. $I=\int\limits_0^1 \frac{x^7}{(1+x^2)^5}dx$	**9.** $I=\int\limits_1^2 \frac{1+x^2}{1+x^4}dx$
2. $I=\int\limits_0^1 x^5(1-x^3)^6dx$	**10.** $I=\int\limits_1^2 \frac{1-x^2}{1+x^4}dx$
3. $I=\int\limits_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)}dx$	**11.** $I=\int\limits_1^2 \frac{1-x^2}{x+x^3}dx$
4. $I=\int\limits_1^2 \frac{dx}{x.(x^{10}+1)^2}$	**12.** $I=\int\limits_0^1 \frac{x^4+1}{x^6+1}dx$
5. $I=\int\limits_1^2 \frac{1-x^7}{x(1+x^7)}dx$	**13.** $I=\int\limits_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{x^4-1}dx$
6. $I=\int\limits_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)}$	**14.** $I=\int\limits_0^1 \frac{xdx}{x^4+x^2+1}$
7. $I=\int\limits_0^1 \frac{(x-1)^2}{(2x+1)^4}dx$	**15.** $I=\int\limits_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1}dx$
8. $I=\int\limits_0^1 \frac{(7x-1)^{99}}{(2x+1)^{101}}dx$	

Bài giải

1. $I=\int\limits_0^1 \frac{x^7}{(1+x^2)^5}dx=\int\limits_0^1 \frac{\left(x^2\right)^3 xdx}{(1+x^2)^5}$

Đặt $t=1+x^2 \Rightarrow dt=2xdx$

Đổi cận: Với $x=0 \Rightarrow t=1$; *Với* $x=1 \Rightarrow t=2$

$$\Rightarrow I=\frac{1}{2}\int\limits_1^2 \frac{(t-1)^3}{t^5}dt=\frac{1}{4}.\frac{1}{2^5}$$

2. $I=\int\limits_0^1 x^5(1-x^3)^6dx=\int\limits_0^1 x^3(1-x^3)x^2dx$

Đặt $t=1-x^3 \Rightarrow dt=-3x^2dx \Rightarrow x^2dx=-\frac{dt}{3}$

Đổi cận: Với $x=0 \Rightarrow t=1$; *Với* $x=1 \Rightarrow t=0$

$$\Rightarrow I=\frac{1}{3}\int\limits_0^1 t^6(1-t)dt=\frac{1}{3}\left(\frac{t^7}{7}-\frac{t^8}{8}\right)=\frac{1}{168}$$

3. $I=\int\limits_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)}dx=\int\limits_1^{\sqrt[4]{3}} \frac{x^3dx}{x^4(x^4+1)}$

Đặt $t = x^4 \Rightarrow dt = 4x^3dx \Rightarrow x^3dx = \dfrac{dt}{4}$

Đổi cận: Với $x = 1 \Rightarrow t = 1;$ *Với* $x = \sqrt[4]{3} \Rightarrow t = 3$

$$\Rightarrow I = \frac{1}{4}\int_1^3 \frac{dt}{t(t+1)} = \frac{1}{4}\int_1^3 \left(\frac{1}{t} - \frac{1}{t+1}\right)dt = \frac{1}{4}\ln\left(\frac{t}{t+1}\right)\Bigg|_1^3 = \frac{1}{4}\ln\frac{3}{2}$$

4. $I = \displaystyle\int_1^2 \frac{dx}{x.(x^{10}+1)^2} = \int_1^2 \frac{x^9dx}{x^{10}(x^{10}+1)^2}$

Đặt $t = x^{10} + 1 \Rightarrow dt = 10x^9dx \Rightarrow x^9dx = \dfrac{dt}{10}$

Đổi cận: Với $x = 1 \Rightarrow t = 2$ *; Với* $x = 2 \Rightarrow t = 2^{10} + 1$

$$\Rightarrow I = \frac{1}{5}\int_2^{2^{10}+1} \frac{dt}{(t-1)t^2} = \frac{1}{5}\int_2^{2^{10}+1}\left(\frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2}\right)dt$$

$$= \frac{1}{5}\left(\ln(t-1) - \ln t + \frac{1}{t}\right)\Bigg|_2^{2^{10}+1} = \frac{1}{5}(10\ln 2 - \ln(2^{10}+1) + \frac{1}{2^{10}+1}) - \frac{1}{5}(-\ln 2 + \frac{1}{2})$$

5. $I = \displaystyle\int_1^2 \frac{1-x^7}{x(1+x^7)}dx = \int_1^2 \frac{(1-x^7).x^6}{x^7.(1+x^7)}dx$.

Đặt $t = x^7 \Rightarrow dt = 7x^6dx \Rightarrow x^6dx = \dfrac{dt}{7}$

Đổi cận: Với $x = 1 \Rightarrow t = 1;$ Với $x = 2 \Rightarrow t = 128$

$$\Rightarrow I = \frac{1}{7}\int_1^{128} \frac{1-t}{t(1+t)}dt = \frac{1}{7}\int_1^{128}\left(\frac{1}{t} - \frac{2}{1+t}\right)dt = \frac{1}{7}(\ln|t| - 2\ln|1+t|)\Big|_1^{128}$$

$$= \frac{1}{7}(7\ln 2 - 2\ln 129) - \frac{1}{7}(-2\ln 2) = \frac{10}{7}\ln 2 - \frac{2}{7}\ln 129$$

6. $I = \displaystyle\int_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)} = \int_1^{\sqrt{3}} \frac{dx}{x^2.x^6(\frac{1}{x^2}+1)}$

Đặt $t = \dfrac{1}{x} \Rightarrow dt = -\dfrac{1}{x^2}dx$

: Đổi cận:Với $x = 1 \Rightarrow t = 1;$ *Với* $x = \sqrt{3} \Rightarrow t = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow I = -\int_1^{\frac{\sqrt{3}}{3}} \frac{t^6}{t^2+1}dt = \int_{\frac{\sqrt{3}}{3}}^1 \left(t^4 - t^2 + 1 - \frac{1}{t^2+1}\right)dt = \frac{117 - 41\sqrt{3}}{135} + \frac{\pi}{12}$$

7. $I = \displaystyle\int_0^1 \frac{(x-1)^2}{(2x+1)^4}dx = \int_0^1 \left(\frac{x-1}{2x+1}\right)^2 \frac{dx}{(2x+1)^2}$

Chú ý: $\left(\dfrac{x-1}{2x+1}\right)' = \dfrac{3}{(2x+1)^2}$

Đặt: $\dfrac{x-1}{2x+1} = t \Rightarrow \dfrac{3dx}{(2x+1)^2} = dt \Rightarrow \dfrac{dx}{(2x+1)^2} = \dfrac{dt}{3}$

Đổi cận: Với: $x = 0 \Rightarrow t = -1$; Với $x = 1 \Rightarrow t = 0$

$$\Rightarrow t = \frac{1}{3}\int_0^{-1} t^2 dt = \frac{t^3}{9}\Big|_0^{-1} = -\frac{1}{9}$$

8. $I = \displaystyle\int_0^1 \left(\frac{7x-1}{2x+1}\right)^{99} \frac{dx}{(2x+1)^2} = \frac{1}{9}\int_0^1 \left(\frac{7x-1}{2x+1}\right)^{99} d\left(\frac{7x-1}{2x+1}\right)$

$$= \frac{1}{9}\cdot\frac{1}{100}\left(\frac{7x-1}{2x+1}\right)^{100}\Bigg|_0^1 = \frac{1}{900}\left[2^{100}-1\right]$$

9. $I = \displaystyle\int_1^2 \frac{1+x^2}{1+x^4}dx$

Ta có: $\dfrac{1+x^2}{1+x^4} = \dfrac{1+\dfrac{1}{x^2}}{x^2+\dfrac{1}{x^2}}$.

Đặt $t = x - \dfrac{1}{x} \Rightarrow dt = \left(1+\dfrac{1}{x^2}\right)dx$

Đổi cận: Với $x = 1 \Rightarrow t = 0$; *Với* $x = 2 \Rightarrow t = \dfrac{3}{2}$

$$\Rightarrow I = \int_0^{\frac{3}{2}} \frac{dt}{t^2-2} = \frac{1}{2\sqrt{2}}\int_0^{\frac{3}{2}}\left(\frac{1}{t-\sqrt{2}} - \frac{1}{t+\sqrt{2}}\right)dt = \frac{1}{2\sqrt{2}}.\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|\Bigg|_0^{\frac{3}{2}} = \frac{1}{\sqrt{2}}\ln(3-2\sqrt{2})$$

10. $I = \displaystyle\int_1^2 \frac{1-x^2}{1+x^4}dx$

Ta có: $\dfrac{1-x^2}{1+x^4} = \dfrac{\dfrac{1}{x^2}-1}{x^2+\dfrac{1}{x^2}}$.

Đặt $t = x + \dfrac{1}{x} \Rightarrow dt = \left(1-\dfrac{1}{x^2}\right)dx$

Đổi cận: Với $x = 1 \Rightarrow t = 2$; *Với* $x = 2 \Rightarrow t = \dfrac{5}{2}$

$$\Rightarrow I = -\int_2^{\frac{5}{2}} \frac{dt}{t^2+2}.$$

Đặt $t = \sqrt{2}\tan u \Rightarrow dt = \sqrt{2}\dfrac{du}{\cos^2 u}$; $\tan u = 2 \Rightarrow u_1 = \arctan 2;\quad \tan u = \dfrac{5}{2} \Rightarrow u_2 = \arctan\dfrac{5}{2}$

$$\Rightarrow I = \frac{\sqrt{2}}{2}\int_{u_1}^{u_2} du = \frac{\sqrt{2}}{2}(u_2 - u_1) = \frac{\sqrt{2}}{2}\left(\arctan\frac{5}{2} - \arctan 2\right)$$

11. $I = \displaystyle\int_1^2 \frac{1-x^2}{x+x^3}dx$

Ta có: $I = \displaystyle\int_1^2 \frac{\frac{1}{x^2}-1}{\frac{1}{x}+x}dx$. *Đặt* $t = x + \dfrac{1}{x} \Rightarrow dt = \left(1 - \dfrac{1}{x^2}\right)dx$

Đổi cận: Với $x = 1 \Rightarrow t = 2;$ *Với* $x = 2 \Rightarrow t = \dfrac{5}{2}$

$$I = -\int_2^{\frac{5}{2}} \frac{dt}{t} = -\ln t\Big|_2^{\frac{5}{2}} = -\ln\frac{5}{2} + \ln 2 = \ln\frac{4}{5}$$

12. $I = \displaystyle\int_0^1 \frac{x^4+1}{x^6+1}dx$

Ta có: $\dfrac{x^4+1}{x^6+1} = \dfrac{(x^4-x^2+1)+x^2}{x^6+1} = \dfrac{x^4-x^2+1}{(x^2+1)(x^4-x^2+1)} + \dfrac{x^2}{x^6+1} = \dfrac{1}{x^2+1} + \dfrac{x^2}{x^6+1}$

$$\Rightarrow I = \int_0^1 \frac{1}{x^2+1}dx + \frac{1}{3}\int_0^1 \frac{d(x^3)}{(x^3)^2+1}dx = \frac{\pi}{4} + \frac{1}{3}.\frac{\pi}{4} = \frac{\pi}{3}$$

13. $I = \displaystyle\int_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{x^4-1}dx$

$$I = \int_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{(x^2-1)(x^2+1)}dx = \frac{1}{2}\int_0^{\frac{\sqrt{3}}{3}}\left(\frac{1}{x^2-1} + \frac{1}{x^2+1}\right)dx = \frac{1}{4}\ln(2-\sqrt{3}) + \frac{\pi}{12}$$

14. $I = \displaystyle\int_0^1 \frac{xdx}{x^4+x^2+1}$.

Đặt $t = x^2 \Rightarrow dt = 2xdx \Rightarrow xdx = \dfrac{dt}{2}$

Đổi cận: $x = 0 \Rightarrow t = 0$; *Với* $x = 1 \Rightarrow t = 1$

$$\Rightarrow I = \frac{1}{2}\int_0^1 \frac{dt}{t^2 + t + 1} = \frac{1}{2}\int_0^1 \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\pi}{6\sqrt{3}}$$

15. $I = \displaystyle\int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2 + 1}{x^4 - x^2 + 1}dx$

Ta có: $\dfrac{x^2 + 1}{x^4 - x^2 + 1} = \dfrac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 1}$.

Đặt $t = x - \dfrac{1}{x} \Rightarrow dt = \left(1 + \dfrac{1}{x^2}\right)dx$

$$\Rightarrow I = \int_0^1 \frac{dt}{t^2 + 1}.$$

Đặt $t = \tan u \Rightarrow dt = \dfrac{du}{\cos^2 u} \Rightarrow I = \displaystyle\int_0^{\frac{\pi}{4}} du = \frac{\pi}{4}$

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PHẦN III TÍCH PHÂN HÀM SỐ VÔ TỶ

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HT 1. Tính các tích phân sau:

a) $I_1 = \int\limits_0^{\sqrt{3}} \frac{xdx}{\sqrt{x^2+1}}$ b) $I_2 = \int\limits_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2+1}}$ c) $I_3 = \int\limits_0^{\sqrt{3}} \sqrt{x^2+1}dx$

Bài giải

a) $I_1 = \int\limits_0^{\sqrt{3}} \frac{xdx}{\sqrt{x^2+1}} = \frac{1}{2}\int\limits_0^{\sqrt{3}} \frac{d(x^2+1)}{\sqrt{x^2+1}} = \sqrt{x^2+1}\Big|_0^1 = \sqrt{2}$

b) $I_2 = \int\limits_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2+1}}$

Đặt: $x + \sqrt{x^2+1} = t \Rightarrow (1 + \frac{x}{\sqrt{x^2+1}})dx = dt \Leftrightarrow \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}dx = dt \Leftrightarrow \frac{dx}{\sqrt{x^2+1}} = \frac{dt}{t}$

Đổi cận: $x = 0 \Rightarrow t = 1; x = \sqrt{3} \Rightarrow t = \sqrt{3}+2$

$\Rightarrow I_2 = \int\limits_1^{\sqrt{3}+2} \frac{dt}{t} = \ln|t|\Big|_1^{\sqrt{3}+2} = \ln(\sqrt{3}+2)$

c) $I_3 = \int\limits_0^{\sqrt{3}} \sqrt{x^2+1}dx$

Đặt: $\begin{cases} u = \sqrt{x^2+1} \\ dv = dx \end{cases} \Rightarrow \begin{cases} du = \frac{x}{\sqrt{x^2+1}}dx \\ v = x \end{cases}$

$\Rightarrow I_3 = x\sqrt{x^2+1}\Big|_0^{\sqrt{3}} - \int\limits_0^{\sqrt{3}} \frac{x^2dx}{\sqrt{x^2+1}} = 2\sqrt{3} - \int\limits_0^{\sqrt{3}} \frac{x^2+1-1}{\sqrt{x^2+1}}dx$

$= 2\sqrt{3} - \int\limits_0^{\sqrt{3}} \sqrt{x^2+1}dx + \int\limits_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2+1}} = 2\sqrt{3} - I_3 + I_2 = 2\sqrt{3} - I_3 + \ln(\sqrt{3}+2)$

$\Rightarrow 2I_3 = 2\sqrt{3} + \ln(\sqrt{3}+2) \Rightarrow I_3 = \sqrt{3} + \frac{1}{2}\ln(\sqrt{3}+2)$

HT 2.Tính các tích phân sau:

a) $I=\int\limits_0^1 x^3\sqrt{1-x^2}dx$ b) $I=\int\limits_{-1}^0 x.\sqrt[3]{x+1}dx$ c) $I=\int\limits_0^1 (x-1)^3\sqrt{2x-x^2}dx$

Bài giải

a) $I=\int\limits_0^1 x^3\sqrt{1-x^2}dx=\int\limits_0^1 x^2\sqrt{1-x^2}xdx$

Đặt: $t=\sqrt{1-x^2}\ \ (t\ge 0)\Leftrightarrow x^2=1-t^2\Rightarrow xdx=-tdt$

Đổi cận: $x=0\Rightarrow t=1;x=1\Rightarrow t=0$

$$\Rightarrow I=-\int\limits_1^0(1-t^2)t.tdt=\int\limits_0^1(t^2-t^4)dt=\left(\frac{t^3}{3}-\frac{t^5}{5}\right)\Bigg|_0^1=\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$$

b) $I=\int\limits_{-1}^0 x.\sqrt[3]{x+1}dx$

Đặt $t=\sqrt[3]{x+1}\Rightarrow t^3=x+1\Rightarrow dx=3t^2dt$

Đổi cận: $x=-1\Rightarrow t=0;x=0\Rightarrow t=1$

$$\Rightarrow I=\int\limits_0^1 3(t^3-1)dt=3\left(\frac{t^7}{7}-\frac{t^4}{4}\right)\Bigg|_0^1=-\frac{9}{28}$$

c) $I=\int\limits_0^1 (x-1)^3\sqrt{2x-x^2}dx$

$$I=\int\limits_0^1(x-1)^3\sqrt{2x-x^2}dx=\int\limits_0^1(x^2-2x+1)\sqrt{2x-x^2}(x-1)dx.$$

Đặt $t=\sqrt{2x-x^2}\Leftrightarrow t^2=2x-x^2\Rightarrow 2tdt=(2-2x)dx\Leftrightarrow(x-1)dx=-tdt$

Đổi cận: $x=0\Rightarrow t=0;x=1\Rightarrow t=1$

$$\Rightarrow I=-\int\limits_0^1(-t^2+1)t.tdt=\int\limits_0^1(t^4-t^2)dt=\left(\frac{t^5}{5}-\frac{t^3}{3}\right)\Bigg|_0^1=\frac{1}{5}-\frac{1}{3}=-\frac{2}{15}.$$

HT 3. Tính các tích phân sau:

a) $I=\int_0^4 \frac{\sqrt{2x+1}}{1+\sqrt{2x+1}}dx$ b) $I=\int_2^6 \frac{dx}{2x+1+\sqrt{4x+1}}$ c) $I=\int_0^1 \frac{1+x}{1+\sqrt{x}}dx$

d) $I=\int_0^3 \frac{x-3}{3\sqrt{x+1}+x+3}dx$ e) $I=\int_1^5 \frac{x^2+1}{x\sqrt{3x+1}}dx$ f) $I=\int_0^3 \frac{2x^2+x-1}{\sqrt{x+1}}dx$

g) $I=\int_0^1 \frac{x^2dx}{(x+1)\sqrt{x+1}}$ h) $I=\int_0^4 \frac{x+1}{\left(1+\sqrt{1+2x}\right)^2}dx$ i) $I=\int_0^2 \frac{2x^3-3x^2+x}{\sqrt{x^2-x+1}}dx$

j) $I=\int_0^2 \frac{x^3dx}{\sqrt[3]{4+x^2}}$ k) $I=\int_1^2 \frac{\sqrt{4-x^2}}{x}dx$ l) $I=\int_2^{2\sqrt{5}} \frac{x}{(x^2+1)\sqrt{x^2+5}}dx$

m) $I=\int_1^{27} \frac{\sqrt{x}-2}{x+\sqrt[3]{x^2}}dx$ o) $I=\int_{\sqrt{3}}^{\sqrt{8}} \frac{x-1}{\sqrt{x^2+1}}dx$ p) $I=\int_1^4 \frac{x^2+\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx$

Bài giải

a) $I=\int_0^4 \frac{\sqrt{2x+1}}{1+\sqrt{2x+1}}dx$

Đặt $t=\sqrt{2x+1}\Rightarrow t^2=2x+1\Rightarrow 2tdt=2dx\Leftrightarrow dx=tdt$

Đổi cận: $x=0\Rightarrow t=1; x=4\Rightarrow t=3$

$$\Rightarrow I=\int_1^3 \frac{t^2}{1+t}dt=\int_1^3\left(t-1+\frac{1}{t+1}\right)dt=\left(\frac{t^2}{2}-t+\ln\left|t+1\right|\right)\Big|_1^3.$$

$$=\left(\frac{9}{2}-3+\ln 4\right)-\left(\frac{1}{2}-1+\ln 2\right)=2+\ln 2$$

b) $I=\int_2^6 \frac{dx}{2x+1+\sqrt{4x+1}}$

Đặt $t=\sqrt{4x+1}\Rightarrow t^2=4x+1\Rightarrow 2tdt=4dx\Rightarrow dx=\frac{tdt}{2}$

Đổi cận: $x=2\Rightarrow t=3; x=6\Rightarrow t=5$

$$\Rightarrow I=\frac{1}{2}\int_3^5 \frac{tdt}{\frac{t^2-1}{2}+1+t}=\int_3^5 \frac{tdt}{t^2+2t+1}=\int_3^5 \frac{tdt}{(t+1)^2}=\int_3^5\left(\frac{1}{t+1}-\frac{1}{(t+1)^2}\right)dt$$

$$=\left(\ln\left|t+1\right|+\frac{1}{t+1}\right)\Big|_3^5=(\ln 6+\frac{1}{6})-(\ln 4+\frac{1}{4})=\ln\frac{3}{2}-\frac{1}{12}$$

c) $I=\int_0^1 \frac{1+x}{1+\sqrt{x}}dx$

Đặt $t=1+\sqrt{x}\Rightarrow x=(t-1)^2\Rightarrow dx=2(t-1)dt$

Đổi cận: $x=0\Rightarrow t=1; x=1\Rightarrow t=2$

$$\Rightarrow I=\int_1^2 \frac{1+(t-1)^2}{t}dt=\int_1^2\left(t-2+\frac{2}{t}\right)dt=\left(\frac{t^2}{2}-2t+2\ln|t|\right)\Big|_1^2=\frac{11}{3}-4\ln 2.$$

d) $I=\int_0^3 \frac{x-3}{3\sqrt{x+1}+x+3}dx$

Đặt $t=\sqrt{x+1}\Rightarrow 2tdt=dx$

Đổi cận: $x=0\Rightarrow t=1; x=3\Rightarrow t=2$

$$\Rightarrow I=\int_1^2 \frac{2t^3-8t}{t^2+3t+2}dt=\int_1^2(2t-6)dt+6\int_1^2\frac{1}{t+1}dt=-3+6\ln\frac{3}{2}$$

e) $I=\int_1^5 \frac{x^2+1}{x\sqrt{3x+1}}dx$

Đặt $t=\sqrt{3x+1}\ \Rightarrow dx=\frac{2tdt}{3}$

Đổi cận: $x=1\Rightarrow t=2; x=5\Rightarrow t=4$

$$\Rightarrow I=\int_2^4 \frac{\left(\frac{t^2-1}{3}\right)^2+1}{\frac{t^2-1}{3}.t}.\frac{2tdt}{3}=\frac{2}{9}\int_2^4(t^2-1)dt+2\int_2^4\frac{dt}{t^2-1}=\frac{2}{9}\int_2^4(t^2-1)dt+2\int_2^4\left(\frac{1}{t-1}-\frac{1}{t+1}\right)dt$$

$$=\frac{2}{9}\left(\frac{1}{3}t^3-t\right)\Bigg|_2^4+\ln\left|\frac{t-1}{t+1}\right|\Bigg|_2^4=\frac{100}{27}+\ln\frac{9}{5}.$$

f) $I=\int_0^3 \frac{2x^2+x-1}{\sqrt{x+1}}dx$

Đặt $\sqrt{x+1}=t\Leftrightarrow x=t^2-1\Rightarrow dx=2tdt$

Đổi cận: $x=0\Rightarrow t=1; x=3\Rightarrow t=2$

$$\Rightarrow I=\int_1^2 \frac{2(t^2-1)^2+(t^2-1)-1}{t}2tdt=2\int_1^2(2t^4-3t^2)dt=\left(\frac{4t^5}{5}-2t^3\right)\Bigg|_1^2=\frac{54}{5}$$

g) $I=\int_0^1 \frac{x^2dx}{(x+1)\sqrt{x+1}}$

Đặt $t = \sqrt{x+1} \Rightarrow t^2 = x+1 \Rightarrow 2tdt = dx$

Đổi cận: $x = 0 \Rightarrow t = 1; x = 1 \Rightarrow t = \sqrt{2}$

$$\Rightarrow I = \int_1^{\sqrt{2}} \frac{(t^2-1)^2}{t^3}.2tdt = 2\int_1^{\sqrt{2}} \left(t - \frac{1}{t}\right)^2 dt = 2\left(\frac{t^3}{3} - 2t - \frac{1}{t}\right)\Bigg|_1^{\sqrt{2}} = \frac{16 - 11\sqrt{2}}{3}$$

h) $I = \int_0^4 \frac{x+1}{\left(1+\sqrt{1+2x}\right)^2} dx$

Đặt $t = 1 + \sqrt{1+2x} \Rightarrow dt = \frac{dx}{\sqrt{1+2x}} \Rightarrow dx = (t-1)dt$ *và* $x = \frac{t^2-2t}{2}$

Đổi cận : $x = 0 \Rightarrow t = 2; x = 4 \Rightarrow t = 4$

$$\Rightarrow I = \frac{1}{2}\int_2^4 \frac{(t^2-2t+2)(t-1)}{t^2} dt = \frac{1}{2}\int_2^4 \frac{t^3-3t^2+4t-2}{t^2} dt = \frac{1}{2}\int_2^4 \left(t - 3 + \frac{4}{t} - \frac{2}{t^2}\right) dt$$

$$= \frac{1}{2}\left(\frac{t^2}{2} - 3t + 4\ln|t| + \frac{2}{t}\right)\Bigg| = 2\ln 2 - \frac{1}{4}$$

i) $I = \int_0^2 \frac{2x^3-3x^2+x}{\sqrt{x^2-x+1}} dx = \int_0^2 \frac{(x^2-x)(2x-1)}{\sqrt{x^2-x+1}} dx$

Đặt $t = \sqrt{x^2-x+1} \Rightarrow 2tdt = (2x-1)dx$

Đổi cận: $x = 0 \Rightarrow t = 1; x = 2 \Rightarrow t = \sqrt{3}$

$$\Rightarrow I = 2\int_1^{\sqrt{3}} (t^2-1)dt = \frac{4}{3}.$$

j) $I = \int_0^2 \frac{x^3 dx}{\sqrt[3]{4+x^2}}$

Đặt $t = \sqrt[3]{4+x^2} \Rightarrow x^2 = t^3 - 4 \Rightarrow 2xdx = 3t^2dt$

Đổi cận: $x = 0 \Rightarrow t = \sqrt[3]{4}; x = 2 \Rightarrow t = 2$

$$\Rightarrow I = \frac{3}{2}\int_{\sqrt[3]{4}}^2 (t^4 - 4t)dt = \frac{3}{2}\left(\frac{t^5}{5} - 2t^2\right)\Bigg|_{\sqrt[3]{4}}^2 = -\frac{3}{2}\left(\frac{8}{5} + 4\sqrt[3]{2}\right)$$

k) $I=\int\limits_1^2 \frac{\sqrt{4-x^2}}{x}dx$

Ta có: $I=\int\limits_1^2 \frac{\sqrt{4-x^2}}{x^2}xdx$.

Đặt t = $\sqrt{4-x^2} \Rightarrow t^2=4-x^2 \Rightarrow tdt=-xdx$

Đổi cận: $x=1\Rightarrow t=\sqrt{3}; x=2\Rightarrow t=0$

$$\Rightarrow I=\int\limits_{\sqrt{3}}^0 \frac{t(-tdt)}{4-t^2}=\int\limits_{\sqrt{3}}^0 \frac{t^2}{t^2-4}dt=\int\limits_{\sqrt{3}}^0 (1+\frac{4}{t^2-4})dt=\left(t+\ln\left|\frac{t-2}{t+2}\right|\right)\Bigg|_{\sqrt{3}}^0 = -\left(\sqrt{3}+\ln\left|\frac{2-\sqrt{3}}{2+\sqrt{3}}\right|\right)$$

l) $I=\int\limits_2^{2\sqrt{5}} \frac{x}{(x^2+1)\sqrt{x^2+5}}dx$

Đặt $t=\sqrt{x^2+5} \Rightarrow t^2=x^2+5\Rightarrow tdt=xdx$

Đổi cận: $x=2\Rightarrow t=3; x=2\sqrt{5}\Rightarrow t=5$

$$I=\int\limits_3^5 \frac{dt}{t^2-4}=\frac{1}{4}\int\limits_3^5\left(\frac{1}{t-2}-\frac{1}{t+2}\right)dt=\frac{1}{4}\ln\frac{15}{7}.$$

m) $I=\int\limits_1^{27}\frac{\sqrt{x}-2}{x+\sqrt[3]{x^2}}dx$

Đặt $t=\sqrt[6]{x}\Rightarrow t^6=x\Rightarrow dx=6t^5dt$

Đổi cận: $x=1\Rightarrow t=1; x=27\Rightarrow t=\sqrt{3}$

$$\Rightarrow I=5\int\limits_1^{\sqrt{3}}\frac{t^3-2}{t(t^2+1)}dt=5\int\limits_1^{\sqrt{3}}\left[1-\frac{2}{t}+\frac{2t}{t^2+1}-\frac{1}{t^2+1}\right]dt=5\left(\sqrt{3}-1+\ln\frac{2}{3}\right)-\frac{5\pi}{12}$$

o) $I=\int\limits_{\sqrt{3}}^{\sqrt{8}}\frac{x-1}{\sqrt{x^2+1}}dx$

$$I=\int\limits_{\sqrt{3}}^{\sqrt{8}}\left(\frac{x}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\right)dx=\frac{1}{2}\int\limits_{\sqrt{3}}^{\sqrt{8}}\frac{d(x^2+1)}{\sqrt{x^2+1}}-\int\limits_{\sqrt{3}}^{\sqrt{8}}\frac{dx}{\sqrt{x^2+1}}$$

$$=\left[\sqrt{x^2+1}-\ln\left(x+\sqrt{x^2+1}\right)\right]\Bigg|_{\sqrt{3}}^{\sqrt{8}} = 1+\ln\left(\sqrt{3}+2\right)-\ln\left(\sqrt{8}+3\right)$$

p) $I=\int\limits_0^4\frac{x^2+\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx$

$$\int_1^4 \frac{x^2+\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx = \int_0^4 \frac{x^2}{\sqrt{1+x\sqrt{x}}}dx + \int_0^4 \frac{\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx$$

+ $I_1 = \int_0^4 \frac{x^2}{\sqrt{1+x\sqrt{x}}}dx$.

Đặt $t=\sqrt{1+x\sqrt{x}} \Leftrightarrow t^2-1 = x\sqrt{x} \Leftrightarrow x^3 = (t^2-1)^2 \Leftrightarrow x^2dx = \frac{4}{3}t(t^2-1)dt$

Đổi cận: $x=0 \Rightarrow t=1; x=4 \Rightarrow t=3$

$$\Rightarrow \int_1^3 \frac{4}{3}(t^2-1)dt = \left(\frac{4}{9}t^3 - \frac{4}{3}t\right)\Bigg|_1^3 = \frac{80}{9}$$

+ $I_2 = \int_0^4 \frac{\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx = \frac{2}{3}\int_0^4 \frac{d(1+x\sqrt{x})}{\sqrt{1+x\sqrt{x}}} = \frac{4}{3}\sqrt{1+x\sqrt{x}}\Big|_0^4 = \frac{8}{3}$

Vậy: $I = \frac{104}{9}$

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HT 4.Tính các tích phân sau:

a) $I = \int_{-1}^1 \frac{dx}{1+x+\sqrt{1+x^2}}$ b) $I = \int_{\frac{1}{3}}^1 \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$ c) $I = \int_0^1 \frac{1}{\sqrt{x^2+x+1}}dx$

d) $I = \int_0^3 \frac{x^2}{(1+\sqrt{1+x})^2(2+\sqrt{1+x})^2}dx$ e) $I = \int_0^3 \frac{x^2}{2(x+1)+2\sqrt{x+1}+x\sqrt{x+1}}dx$

f) $I = \int_1^{2\sqrt{2}} \frac{\sqrt[3]{x-x^3}+2011x}{x^4}dx$ g) $I = \int_{\sqrt{3}}^{2\sqrt{2}} \frac{x^4}{\left(x-\frac{1}{x}\right)\sqrt{x^2+1}}dx$ h) $I = \int_{\frac{1}{3}}^{\frac{2}{3}} \frac{x}{3x+\sqrt{9x^2-1}}dx$

Bài giải

a) $I = \int_{-1}^1 \frac{dx}{1+x+\sqrt{1+x^2}}$

Ta có: $I = \int_{-1}^1 \frac{1+x-\sqrt{1+x^2}}{(1+x)^2-(1+x^2)}dx = \int_{-1}^1 \frac{1+x-\sqrt{1+x^2}}{2x}dx = \frac{1}{2}\int_{-1}^1\left(\frac{1}{x}+1\right)dx - \int_{-1}^1 \frac{\sqrt{1+x^2}}{2x}dx$

+ $I_1 = \frac{1}{2}\int\limits_{-1}^{1}\left(\frac{1}{x}+1\right)dx = \frac{1}{2}\left[\ln\left|x\right|+x\right]\Big|_{-1}^{1} = 1$

+ $I_2 = \int\limits_{-1}^{1}\frac{\sqrt{1+x^2}}{2x}dx$. *Đặt* $t=\sqrt{1+x^2} \Rightarrow t^2 = 1+x^2 \Rightarrow 2tdt = 2xdx \Rightarrow$ *I₂=* $\int\limits_{\sqrt{2}}^{\sqrt{2}}\frac{t^2dt}{2(t^2-1)} = 0$

Vậy: $I = 1$.

<u>*Cách 2*</u>*: Đặt* $t = x+\sqrt{x^2+1} \Leftrightarrow t - x = \sqrt{x^2+1} \Rightarrow (t-x)^2 = x^2+1 \Leftrightarrow t^2 - 2tx = 1 \Leftrightarrow x = \frac{t^2-1}{2t}$

$\Rightarrow dx = \left(\frac{1}{2}+\frac{1}{2t^2}\right)dt$

Đổi cận: $x=-1 \Rightarrow t = -1+\sqrt{2}; x = 1 \Rightarrow t = 1+\sqrt{2}$

$$\Rightarrow I = \int\limits_{-1+\sqrt{2}}^{1+\sqrt{2}}\frac{(t^2+1)dx}{2t^2(1+t)} = \frac{1}{2}\int\limits_{-1+\sqrt{2}}^{1+\sqrt{2}}\left(\frac{2}{t+1}+\frac{1}{t^2}-\frac{1}{t}\right)dt$$

$$= \frac{1}{2}\left(2\ln\left|t+1\right| - \frac{1}{t} - \ln\left|t\right|\right)\Bigg|_{-1+\sqrt{2}}^{1+\sqrt{2}} = \frac{1}{2}\left(\ln\frac{(t+1)^2}{t} - \frac{1}{t}\right)\Bigg|_{-1+\sqrt{2}}^{1+\sqrt{2}}$$

$$= \frac{1}{2}(\ln(2+2\sqrt{2})+1-\sqrt{2}) - \frac{1}{2}(\ln(2+2\sqrt{2})-1-\sqrt{2}) = 1$$

b) $I = \int\limits_{\frac{1}{3}}^{1}\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$

Ta có: $I = \int\limits_{\frac{1}{3}}^{1}\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}.\frac{1}{x^3}dx$

Đặt $t = \frac{1}{x^2}-1 \Rightarrow dt = -\frac{2}{x^3}dx \Leftrightarrow \frac{dx}{x^3} = -\frac{dt}{2}$

Đổi cận: $x = \frac{1}{3} \Rightarrow t = 8; x = 1 \Rightarrow t = 0$

$$\Rightarrow I = -\frac{1}{2}\int\limits_{8}^{0}t^{\frac{1}{3}}dt = \frac{1}{2}\int\limits_{0}^{8}t^{\frac{1}{3}}dt = \frac{1}{2}.\frac{3}{4}t^{\frac{4}{3}}\Big|_0^8 = 6$$

c) $I = \int\limits_{0}^{1}\frac{1}{\sqrt{x^2+x+1}}dx = \int\limits_{0}^{1}\frac{dx}{\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}}$

Đặt: $\Rightarrow dt = \left(1 + \dfrac{2x+1}{2\sqrt{x^2+x+1}}\right)dx \Leftrightarrow dt = \left(\dfrac{\sqrt{x^2+x+1}+x+\dfrac{1}{2}}{\sqrt{x^2+x+1}}\right)dx \Rightarrow \dfrac{dx}{\sqrt{x^2+x+1}} = \dfrac{dt}{t}$

Đổi cận: $x = 0 \Rightarrow t = \dfrac{3}{2}; x = 1 \Rightarrow t = \dfrac{3}{2} + \sqrt{3} \quad t = x + \dfrac{1}{2} + \sqrt{x^2+x+1}$

$$I = \int_{\frac{3}{2}}^{\frac{3}{2}+\sqrt{3}} \frac{dt}{t} = \ln|t|\Big|_{\frac{3}{2}}^{\frac{3}{2}+\sqrt{3}} = \ln\left(\frac{3}{2}+\sqrt{3}\right) - \ln\frac{3}{2} = \ln\frac{3+2\sqrt{3}}{3}$$

d) $I = \displaystyle\int_0^3 \frac{x^2}{(1+\sqrt{1+x})^2(2+\sqrt{1+x})^2}dx$

Đặt $2+\sqrt{1+x} = t \Rightarrow t-2 = \sqrt{1+x} \Rightarrow (t-2)^2 = 1+x \Rightarrow 2(t-2)dt = dx$

Đổi cận: $x = 0 \Rightarrow t = 3; x = 3 \Rightarrow t = 4$

$$\Rightarrow I = \int_3^4 \frac{\left((t-2)^2-1\right)^2.2(t-2)dt}{(t-1)^2t^2} = \int_3^4 \frac{(t-1)^2(t-3)^2.2(t-2)dt}{(t-1)^2t^2} = \int_3^4 \frac{2(t-3)^2(t-2)dt}{t^2}$$

$$= \int_3^4 \left(2t - 16 + \frac{42}{t} - \frac{36}{t^2}\right)dt = \left(t^2 - 16t + 42\ln|t| + \frac{36}{t}\right)\Big|_3^4 = -12 + 42\ln\frac{4}{3}$$

e) $I = \displaystyle\int_0^3 \frac{x^2}{2(x+1)+2\sqrt{x+1}+x\sqrt{x+1}}dx$

Đặt: $t = \sqrt{x+1} \Rightarrow t^2 = x+1 \Rightarrow 2tdt = dx$

Đổi cận: $x = 0 \Rightarrow t = 1; x = 3 \Rightarrow t = 2$

$$\Rightarrow I = \int_1^2 \frac{2t(t^2-1)^2dt}{t(t+1)^2} = 2\int_1^2 (t-1)^2dt = \frac{2}{3}(t-1)^3\Big|_1^2 = \frac{2}{3}$$

f) $I = \displaystyle\int_1^{2\sqrt{2}} \frac{\sqrt[3]{x-x^3}+2011x}{x^4}dx$

Ta có: $I = \displaystyle\int_1^{2\sqrt{2}} \frac{\sqrt[3]{\dfrac{1}{x^2}-1}}{x^3}dx + \int_1^{2\sqrt{2}} \frac{2011}{x^3}dx = M + N$

+) $M=\int\limits_{1}^{2\sqrt{2}}\dfrac{\sqrt[3]{\dfrac{1}{x^2}-1}}{x^3}dx$.

Đặt $t=\sqrt[3]{\dfrac{1}{x^2}-1}\Rightarrow t^3=\dfrac{1}{x^2}-1\Rightarrow 3t^2dt=-\dfrac{2}{x^3}dx\Rightarrow\dfrac{dx}{x^3}=-\dfrac{3}{2}t^2dt$

Đổi cận: $x=1\Rightarrow t=0; x=2\sqrt{2}\Rightarrow t=-\dfrac{\sqrt[3]{7}}{2}$

$$\Rightarrow M=-\frac{3}{2}\int\limits_{0}^{-\frac{\sqrt[3]{7}}{2}}t^3dt=-\frac{21\sqrt[3]{7}}{128}$$

+) $N=\int\limits_{1}^{2\sqrt{2}}\dfrac{2011}{x^3}dx=\int\limits_{1}^{2\sqrt{2}}2011x^{-3}dx=\left[-\dfrac{2011}{2x^2}\right]\Bigg|_{1}^{2\sqrt{2}}=\dfrac{14077}{16}$

$\Rightarrow I=\dfrac{14077}{16}-\dfrac{21\sqrt[3]{7}}{128}$.

g) $I=\int\limits_{\sqrt{3}}^{2\sqrt{2}}\dfrac{x^4}{\left(x-\dfrac{1}{x}\right)\sqrt{x^2+1}}dx=\int\limits_{\sqrt{3}}^{2\sqrt{2}}\dfrac{x^4.xdx}{(x^2-1)\sqrt{x^2+1}}$

Đặt $t=\sqrt{x^2+1}\Rightarrow dt=\dfrac{xdx}{\sqrt{x^2+1}}$

Đổi cận: $x=\sqrt{3}\Rightarrow t=2; x=2\sqrt{2}\Rightarrow t=3$

$\Rightarrow I=\int\limits_{2}^{3}\dfrac{(t^2-1)^2}{t^2-2}dt=\int\limits_{2}^{3}\dfrac{t^4-2t^2+1}{t^2-2}dt=\int\limits_{2}^{3}t^2dt+\int\limits_{2}^{3}\dfrac{1}{t^2-2}dt=\dfrac{19}{3}+\dfrac{\sqrt{2}}{4}\ln\left(\dfrac{4+\sqrt{2}}{4-\sqrt{2}}\right)$

h) $I=\int\limits_{\frac{1}{3}}^{\frac{2}{3}}\dfrac{x}{3x+\sqrt{9x^2-1}}dx=\int\limits_{\frac{1}{3}}^{\frac{2}{3}}x(3x-\sqrt{9x^2-1})dx=\int\limits_{\frac{1}{3}}^{\frac{2}{3}}3x^2dx-\int\limits_{\frac{1}{3}}^{\frac{2}{3}}x\sqrt{9x^2-1}dx$

+ $I_1=\int\limits_{\frac{1}{3}}^{\frac{2}{3}}3x^2dx=x^3\Bigg|_{\frac{1}{3}}^{\frac{2}{3}}=\dfrac{8}{27}-\dfrac{1}{27}=\dfrac{7}{27}$

+ $I_2=\int\limits_{\frac{1}{3}}^{\frac{2}{3}}x\sqrt{9x^2-1}dx=\dfrac{1}{18}\int\limits_{\frac{1}{3}}^{\frac{2}{3}}\sqrt{9x^2-1}\,d(9x^2-1)=\dfrac{1}{27}(9x^2-1)^{\frac{3}{2}}\Bigg|_{\frac{1}{3}}^{\frac{2}{3}}=\dfrac{\sqrt{3}}{9}$

$\Rightarrow I = \frac{7 - 3\sqrt{3}}{27}$

HT 5.Tính các tích phân sau:

a) $I = \int_0^2 x^2\sqrt{4 - x^2}dx$ **b)** $I_2 = \int_{-1}^0 \sqrt{-x^2 - 2x}dx$ **c)** $I = \int_0^1 \sqrt{3 + 2x - x^2}dx$

d) $I = \int_0^1 \frac{x^2dx}{\sqrt{4 - x^6}}$ **e)** $I = \int_0^{\frac{1}{2}} \sqrt{1 - 2x\sqrt{1 - x^2}}dx$ **f)** $I = \int_0^1 \frac{x^2dx}{\sqrt{3 + 2x - x^2}}$

Bài giải

a) $I = \int_0^2 x^2\sqrt{4 - x^2}dx$

Đặt: $x = 2\sin t$, Với $t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \ge 0$

$\Rightarrow dx = 2\cos tdt$

Đổi cận: $x = 0 \Rightarrow t = 0; x = 2 \Rightarrow t = \frac{\pi}{2}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} 4\sin^2 t\sqrt{4 - 4\sin^2 t}.2\cos t.dt = 16\int_0^{\frac{\pi}{2}} \sin^2 t\sqrt{1 - \sin^2 t}.\cos t.dt$$

$$= 16\int_0^{\frac{\pi}{2}} \sin^2 t.\left|\cos t\right|\cos tdt = 16\int_0^{\frac{\pi}{2}} \sin^2 t.\cos^2 t.dt = 4\int_0^{\frac{\pi}{2}} \sin^2 4t.dt = 2\int_0^{\frac{\pi}{2}} (1 - \cos 8t)dt$$

$$= 2(t - \frac{\sin 8t}{8})\Big|_0^{\frac{\pi}{2}} = \pi$$

b) $I_2 = \int_{-1}^0 \sqrt{-x^2 - 2x}dx = \int_{-1}^0 \sqrt{1 - (x + 1)^2}dx$

Đặt: $x + 1 = \sin t$, Với $t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \ge 0$

$\Rightarrow dx = \cos tdt$

Đổi cận: $x = -1 \Rightarrow t = 0; x = 0 \Rightarrow t = \frac{\pi}{2}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t}.\cos t.dt = \int_0^{\frac{\pi}{2}} \cos^2 t.dt = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2t)dt$$

$$= \frac{1}{2}\left(t + \frac{\sin 2t}{2}\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

c) $I=\int_0^1\sqrt{3+2x-x^2}dx=\int_0^1\sqrt{4-(x-2)^2}dx$

Đặt: $x-2=2\sin t$, Với $t\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]\Rightarrow \cos t\ge 0$

$\Rightarrow dx=2\cos tdt$

Đổi cận: $x=0\Rightarrow t=-\frac{\pi}{2};x=1\Rightarrow t=-\frac{\pi}{6}$

$\Rightarrow I=\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}\sqrt{4-4\sin^2 t}.2\cos t.dt=4\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}\cos^2 t.dt=2\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}(1+\cos 2t)dt$

$=2\left(t+\frac{\sin 2t}{2}\right)\Bigg|_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}=-\frac{\pi}{12}-\frac{\sqrt{3}}{4}+\frac{\pi}{4}=\frac{\pi}{6}-\frac{\sqrt{3}}{4}$

d) $I=\int_0^1\frac{x^2dx}{\sqrt{4-x^6}}$

Đặt $t=x^3\Rightarrow dt=3x^2dx$

Đổi cận: $x=0\Rightarrow t=0;x=1\Rightarrow t=1$

$\Rightarrow I=\frac{1}{3}\int_0^1\frac{dt}{\sqrt{4-t^2}}.$

Đặt: $t=2\sin u,\ u\in\left[0;\frac{\pi}{2}\right]\Rightarrow dt=2\cos udu$

Đổi cận: $t=0\Rightarrow u=0;t=1\Rightarrow u=\frac{\pi}{6}$

$\Rightarrow I=\frac{1}{3}\int_0^{\frac{\pi}{6}}\frac{2\cos u.du}{\sqrt{4-4\sin^2 u}}=\frac{1}{3}\int_0^{\frac{\pi}{6}}du=\frac{u}{3}\Bigg|_0^{\frac{\pi}{6}}=\frac{\pi}{18}.$

Chú ý: Các em học sinh có thể đặt trực tiếp: $x^3=2\sin t$

e) $I=\int_0^{\frac{1}{2}}\sqrt{1-2x\sqrt{1-x^2}}dx$

Đặt $x=\sin t$, *Với* $t\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]\Rightarrow \cos t\ge 0;\cos t>\sin t$

$\Rightarrow dx=\cos t.dt$

Đổi cận: $x=0\Rightarrow t=0;x=\frac{1}{2}\Rightarrow t=\frac{\pi}{6}$

$$\Rightarrow I=\int_0^{\frac{\pi}{6}}\sqrt{1-2\sin t\sqrt{1-\sin^2 t}}.\cos t.dt=\int_0^{\frac{\pi}{6}}\sqrt{1-2\sin t.\cos t}\cos t.dt=\int_0^{\frac{\pi}{6}}\sqrt{(\sin t-\cos t)^2}\cos t.dt$$

$$=\int_0^{\frac{\pi}{6}}(\cos t-\sin t)\cos tdt=\int_0^{\frac{\pi}{6}}(\cos^2 t-\sin t.\cos t)dt=\frac{1}{2}\int_0^{\frac{\pi}{6}}(1+\cos 2t-\sin 2t)dt=\frac{1}{2}(t+\frac{\sin 2t}{2}+\frac{\cos 2t}{2})\Big|_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{12}+\frac{\sqrt{3}}{8}-\frac{1}{8}$$

f) $I=\int_0^1\frac{x^2dx}{\sqrt{3+2x-x^2}}$

Ta có: $I=\int_0^1\frac{x^2dx}{\sqrt{2^2-(x-1)^2}}$.

Đặt $x-1=2\sin t$. Với $t\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]\Rightarrow\cos t\geq 0$

$\Rightarrow dx=2\cos tdt$

Đổi cận: $x=0\Rightarrow t=-\frac{\pi}{2};x=1\Rightarrow t=-\frac{\pi}{6}$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}\frac{(1+2\sin t)^2 2\cos t}{\sqrt{4-(2\sin t)^2}}dt=\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}\left(1+4\sin t+4\sin^2 t\right)dt=\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}(1+4\sin t+2-2\cos 8t)dt$$

$$=(3t-4\cos t-\frac{\sin 8t}{4})\Big|_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}=\frac{\pi}{2}+\frac{3\sqrt{3}}{2}-4$$

HT 6.Tính các tích phân sau:

a) $I=\int_{-2}^{2}(x^5+x^2)\sqrt{4-x^2}dx$

b) $I=\int_1^2\frac{\left(3-\sqrt{4-x^2}\right)dx}{2x^4}$

c) $I=\int_0^2\sqrt{\frac{2-x}{x+2}}dx$

d) $I=\int_0^1\left(\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}-2x\ln\left(1+x\right)\right)dx$

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Bài giải

a) $I=\int_{-2}^{2}(x^5+x^2)\sqrt{4-x^2}dx$

$$=\int_{-2}^{2}(x^5+x^2)\sqrt{4-x^2}dx=\int_{-2}^{2}x^5\sqrt{4-x^2}dx+\int_{-2}^{2}x^2\sqrt{4-x^2}dx=A+B.$$

+ Tính A = $\int\limits_{-2}^{2} x^5\sqrt{4-x^2}\,dx = \int\limits_{-2}^{2} x^4\sqrt{4-x^2}\,xdx$.

Đặt $t = \sqrt{4-x^2} \Rightarrow t^2 = 4-x^2 \Rightarrow xdx = -tdt$

Đổi cận: $x=-2 \Rightarrow t=0; x=2 \Rightarrow t=0$

$$\Rightarrow I = \int\limits_0^0 (4-t^2)^2.t^2.dt = 0$$

+ Tính B = $\int\limits_{-2}^{2} x^2\sqrt{4-x^2}\,dx$.

Đặt: $x = 2\sin t$, Với $t \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right] \Rightarrow \cos t \geq 0$

$\Rightarrow dx = 2\cos tdt$

Đổi cận: $x=-2 \Rightarrow t = -\frac{\pi}{2}; x=2 \Rightarrow t=\frac{\pi}{2}$

$$\Rightarrow B = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\sin^2 t\sqrt{4-4\sin^2 t}.2\cos t.dt = 16\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t\sqrt{1-\sin^2 t}.\cos t.dt$$

$$= 16\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t.\left|\cos t\right|\cos tdt = 16\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t.\cos^2 t.dt = 4\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 4t.dt = 2\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1-\cos 8t)dt$$

$$= 2(t - \frac{\sin 8t}{8})\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2\pi$$

Vậy, $I = 2\pi$

b) $I = \int\limits_1^2 \frac{\left(3-\sqrt{4-x^2}\right)dx}{2x^4}$

Ta có: $I = \int\limits_1^2 \frac{3}{2x^4}dx - \int\limits_1^2 \frac{\sqrt{4-x^2}}{2x^4}dx$.

+ Tính $I_1 = \int\limits_1^2 \frac{3}{2x^4}dx = \frac{3}{2}\int\limits_1^2 x^{-4}dx = \frac{7}{16}$.

+ Tính $I_2 = \int\limits_1^2 \frac{\sqrt{4-x^2}}{2x^4}dx$.

Đặt $x = 2\sin t \Rightarrow dx = 2\cos tdt$.

Đổi cận: $x = 1 \Rightarrow t = \frac{\pi}{6}; x = 2 \Rightarrow t = \frac{\pi}{2}$

$$\Rightarrow I_2 = \frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^2 t dt}{\sin^4 t} = \frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t\left(\frac{1}{\sin^2 t}\right)dt = -\frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t.d(\cot t) = \frac{\sqrt{3}}{8}$$

Vậy: $I = \frac{1}{16}\left(7 - 2\sqrt{3}\right)$.

c) $I = \int_0^2 \sqrt{\frac{2-x}{x+2}}dx$

Đặt $x = 2\cos t \Rightarrow dx = -2\sin t dt$

Đổi cận: $x = 0 \Rightarrow t = \frac{\pi}{2}; x = 2 \Rightarrow t = 0$

$$\Rightarrow I = -\int_{\frac{\pi}{2}}^{0} \sqrt{\frac{2-2\cos t}{2+2\cos t}}2\sin t dt = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 \frac{t}{2}}{\cos^2 \frac{t}{2}}}2\sin t.dt.$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}4.\sin\frac{t}{2}.\cos\frac{t}{2}.dt = \int_0^{\frac{\pi}{2}} 2(1-\cos t)dt$$

$$= 2(t - \sin t)\Big|_0^{\frac{\pi}{2}} = \pi - 2$$

d) $I = \int_0^1 \left(\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} - 2x\ln\left(1+x\right)\right)dx$

Tính $H = \int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$. *Đặt* $\sqrt{x} = \cos t; t \in \left[0; \frac{\pi}{2}\right] \Rightarrow H = 2 - \frac{\pi}{2}$

Tính: $K = \int_0^1 2x\ln(1+x)dx$. *Đặt* $\begin{cases} u = \ln(1+x) \\ dv = 2xdx \end{cases} \Rightarrow K = \frac{1}{2}$

Vậy: $I = \frac{3}{2} - \frac{\pi}{2}$

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PHẦN IV TÍCH PHÂN HÀM LƯỢNG GIÁC

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HT 1.Tính các tích phân sau:

a) $I = \int\limits_0^{\pi} \cos^4 x dx$ **b)** $I_2 = \int\limits_0^{\frac{\pi}{2}} (\sin^6 x + \cos^6 x)dx$ **c)** $I = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x.\sin 5x.dx$

d) $I = \int\limits_0^{\frac{\pi}{2}} \frac{4\sin^3 x}{1+\cos x}dx$ **e)** $I = \int\limits_0^{\frac{\pi}{3}} \frac{\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)}{\cos x}dx$ **f)** $I = \int\limits_0^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}$

Bài giải

a) $I = \int\limits_0^{\pi} \cos^4 x dx = \int\limits_0^{\pi} (\cos^2 x)^2 dx = \int\limits_0^{\pi} \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int\limits_0^{\pi} (1 + 2\cos 2x + \cos^2 2x)dx$

$= \frac{1}{4}\int\limits_0^{\pi} \left(\frac{3}{2} + 2\cos 2x + \frac{\cos 4x}{2}\right)dx = \frac{1}{4}\left(\frac{3}{2}x + \sin 2x + \frac{\sin 4x}{8}\right)\Bigg|_0^{\pi} = \frac{3\pi}{8}$

b) $I_2 = \int\limits_0^{\frac{\pi}{2}} (\sin^6 x + \cos^6 x)dx = \int\limits_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x)dx$

$= \int\limits_0^{\frac{\pi}{2}} \left((\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x\right)dx = \int\limits_0^{\frac{\pi}{2}} (1 - \frac{3}{4}\sin^2 2x)dx$

$= \int\limits_0^{\frac{\pi}{2}} (\frac{5}{8} + \frac{3}{8}\cos 4x)dx = \left(\frac{5}{8}x + \frac{3}{32}\sin 4x\right)\Bigg|_0^{\frac{\pi}{2}} = \frac{5\pi}{16}$

c) $I = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x.\sin 5x.dx = \frac{1}{2}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos 3x - \cos 7x)dx = \frac{1}{2}\left(\frac{\sin 3x}{3} - \frac{\sin 7x}{7}\right)\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\frac{4}{21}$

d) $I = \int\limits_0^{\frac{\pi}{2}} \frac{4\sin^3 x}{1+\cos x}dx = \int\limits_0^{\frac{\pi}{2}} \frac{4(1-\cos^2 x)\sin x}{1+\cos x}dx = \int\limits_0^{\frac{\pi}{2}} 4(1-\cos x)d(1-\cos x) = 2(1-\cos x)^2\Bigg|_0^{\frac{\pi}{2}} = 2$

e) $I = \int\limits_0^{\frac{\pi}{3}} \frac{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)}{\cos x} dx = \int\limits_0^{\frac{\pi}{3}} \frac{\sin x - \cos x}{\cos x} dx = \int\limits_0^{\frac{\pi}{3}} \left(\frac{\sin x}{\cos x} - 1\right) dx$

$= \left(-\ln\left|\cos x\right| - x\right)\Big|_0^{\frac{\pi}{3}} = \ln 2 - \frac{\pi}{3}$

f) $I = \int\limits_0^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x} = \int\limits_0^{\frac{\pi}{4}} \frac{dx}{2\cos^2 x} = \frac{1}{2}\tan x\Big|_0^{\frac{\pi}{4}} = \frac{1}{2}$

HT 2. Tính các tích phân sau:

a) $\mathrm{I} = \int\limits_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx$ b) $I = \int\limits_0^{\frac{\pi}{2}} (\cos^3 x - 1)\cos^2 x.dx$ c) $I = \int\limits_0^{\frac{\pi}{4}} \frac{dx}{\cos^6 x}$

d) $I = \int\limits_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x)dx$ e) $I = \int\limits_0^{\frac{\pi}{2}} \cos 2x(\sin^4 x + \cos^4 x)dx$

Bài giải

a) $\mathrm{I} = \int\limits_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx$

$$\mathrm{I} = \int\limits_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx = \frac{1}{2}\int\limits_0^{\frac{\pi}{2}} (1+\cos 2x)\cos 2x dx = \frac{1}{4}\int\limits_0^{\frac{\pi}{2}} (1 + 2\cos 2x + \cos 4x)dx$$

$$= \frac{1}{4}\left(x + \sin 2x + \frac{1}{4}\sin 4x\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{8}$$

b) $I = \int\limits_0^{\frac{\pi}{2}} (\cos^3 x - 1)\cos^2 x.dx = \int\limits_0^{\frac{\pi}{2}} (\cos^5 x - \cos^2 x)dx$

$A = \int\limits_0^{\frac{\pi}{2}} \cos^5 x dx = \int\limits_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 d(\sin x) = \frac{8}{15}$

$B = \int\limits_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{1}{2}\int\limits_0^{\frac{\pi}{2}} (1+\cos 2x)dx = \frac{\pi}{4}$

Vậy I = $\frac{8}{15} - \frac{\pi}{4}$.

c) $I = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^6 x} = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x.\cos^2 x} = \int_0^{\frac{\pi}{4}} (1 + 2\tan^2 x + \tan^4 x) d(\tan x) = \frac{28}{15}$.

d) $I = \int_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) dx$.

Ta có: $(\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) = \frac{33}{64} + \frac{7}{16}\cos 4x + \frac{3}{64}\cos 8x$

$\Rightarrow I = \frac{33}{128}\pi$.

e) $I = \int_0^{\frac{\pi}{2}} \cos 2x(\sin^4 x + \cos^4 x) dx$

$$I = \int_0^{\frac{\pi}{2}} \cos 2x\left(1 - \frac{1}{2}\sin^2 2x\right) dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2}\sin^2 2x\right) d(\sin 2x) = 0$$

HT 3.Tính các tích phân sau :

a) $I = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\cot x - \tan x - 2\tan 2x}{\sin 4x} dx$ **b)** $I = \int_0^{\frac{\pi}{6}} \frac{1}{2\sin x - \sqrt{3}} dx$ **c)** $I = \int_{\frac{\pi}{3}}^{\pi} \frac{dx}{2 + \sqrt{3}\sin x - \cos x}$

d) $I = \int \frac{\cos^2\left(x + \frac{\pi}{8}\right)}{\sin 2x + \cos 2x + \sqrt{2}} dx$ **e)** $I = \int \frac{8\cos^2 x - \sin 2x - 3}{\sin x - \cos x} dx$ **f)** $I = \int_0^{2\pi} \sqrt{1 + \sin x} dx$

Bài giải

a) $I = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\cot x - \tan x - 2\tan 2x}{\sin 4x} dx$

Ta có: $I = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{2\cot 2x - 2\tan 2x}{\sin 4x} dx = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{2\cot 4x}{\sin 4x} dx = 2\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\cos 4x}{\sin^2 4x} dx = -\frac{1}{2\sin 4x}\Bigg|_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \frac{2\sqrt{3} - 3}{6}$

b) $I = \int_0^{\frac{\pi}{6}} \frac{1}{2\sin x - \sqrt{3}} dx$

Ta có: $I=\frac{1}{2}\int\limits_0^{\frac{\pi}{6}}\frac{1}{\sin x-\sin\frac{\pi}{3}}dx=\int\limits_0^{\frac{\pi}{6}}\frac{\frac{1}{2}}{\sin x-\sin\frac{\pi}{3}}dx$

$$=\int\limits_0^{\frac{\pi}{6}}\frac{\cos\frac{\pi}{3}}{\sin x-\sin\frac{\pi}{3}}dx=\int\limits_0^{\frac{\pi}{6}}\frac{\cos\left(\left(\frac{x}{2}+\frac{\pi}{6}\right)-\left(\frac{x}{2}-\frac{\pi}{6}\right)\right)}{2\cos\left(\frac{x}{2}+\frac{\pi}{6}\right).\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)}dx$$

$$=\frac{1}{2}\int\limits_0^{\frac{\pi}{6}}\frac{\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)}dx+\frac{1}{2}\int\limits_0^{\frac{\pi}{6}}\frac{\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)}dx=\ln\left|\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)\right|\Big|_0^{\frac{\pi}{6}}-\ln\left|\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)\right|\Big|_0^{\frac{\pi}{6}}=.....$$

c) $I=\int\limits_{\frac{\pi}{3}}^{\pi}\frac{dx}{2+\sqrt{3}\sin x-\cos x}$

$$I=\frac{1}{2}\int\limits_{\frac{\pi}{3}}^{\pi}\frac{dx}{1-\cos\left(x+\frac{\pi}{3}\right)}=I=\frac{1}{4}\int\limits_{\frac{\pi}{3}}^{\pi}\frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right)}=\frac{1}{4\sqrt{3}}.$$

d) $I=\int\frac{\cos^2\left(x+\frac{\pi}{8}\right)}{\sin 2x+\cos 2x+\sqrt{2}}\,dx$

Ta có: $I=\frac{1}{2\sqrt{2}}\int\frac{1+\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx$

$$=\frac{1}{2\sqrt{2}}\left(\int\frac{\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx+\int\frac{dx}{\left[\sin\left(x+\frac{\pi}{8}\right)+\cos\left(x+\frac{\pi}{8}\right)\right]^2}\right)$$

$$=\frac{1}{2\sqrt{2}}\left(\int\frac{\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx+\frac{1}{2}\int\frac{dx}{\sin^2\left(x+\frac{3\pi}{8}\right)}\right)$$

$$=\frac{1}{4\sqrt{2}}\left(\ln\left|1+\sin\left(2x+\frac{\pi}{4}\right)\right|-\cot\left(x+\frac{3\pi}{8}\right)\right)+C$$

e) $I=\int\frac{8\cos^2x-\sin 2x-3}{\sin x-\cos x}dx$

$$I=\int\frac{(\sin x-\cos x)^2+4\cos 2x}{\sin x-\cos x}dx=\int\left[(\sin x-\cos x-4(\sin x+\cos x)\right]dx$$

$= 3\cos x - 5\sin x + C$.

f) $I = \int\limits_0^{2\pi} \sqrt{1+\sin x}dx$

$$I = \int\limits_0^{2\pi} \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}dx = \int\limits_0^{2\pi} \left|\sin\frac{x}{2} + \cos\frac{x}{2}\right|dx = \sqrt{2}\int\limits_0^{2\pi}\left|\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|dx$$

$$= \sqrt{2}\left[\int\limits_0^{\frac{3\pi}{2}} \sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx - \int\limits_{\frac{3\pi}{2}}^{2\pi} \sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx\right] = 4\sqrt{2}$$

HT 4.Tính các tích phân sau:

1. $I = \int\limits_0^{\frac{\pi}{2}} \frac{\sin 2x}{\left(2+\sin x\right)^2}dx$	**9.** $I = \int\limits_0^{\frac{\pi}{2}} \frac{\cos 2x}{\left(\cos x - \sin x + 3\right)^3}dx$
2. $I = \int\limits_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^6 x + \cos^6 x}}dx$	**10.** $I = \int\limits_0^{\frac{\pi}{4}} \frac{\sin 4x}{\cos^2 x.\sqrt{\tan^4 x + 1}}dx$
3. $I = \int\limits_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}dx$	**11.** $I = \int\limits_0^{\frac{\pi}{4}} \frac{\sin 4x}{1+\cos^2 x}dx$
4. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x+\sin x)\sin x}{\sin^3 x + \sin^2 x}dx$	**12.** $I = \int\limits_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos 2x}dx$
5. $I = \int\limits_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{\cos^2 x + 4\sin^2 x}}dx$	**13.** $I = \int\limits_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\sqrt{3-\sin 2x}}dx$
6. $I = \int\limits_0^{\frac{\pi}{6}} \frac{\tan\left(x - \frac{\pi}{4}\right)}{\cos 2x}dx$	**14.** $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x.\sin\left(x+\frac{\pi}{4}\right)}dx$
7. $I = 2\int\limits_1^2 \sqrt[6]{1-\cos^3 x}.\sin x.\cos^5 x dx$	**15.** $I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x.\cos^4 x}$
8. $I = \int\limits_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos x\sqrt{1+\cos^2 x}}$	

Bài giải

1. $I = \int\limits_0^{\frac{\pi}{2}} \frac{\sin 2x}{\left(2+\sin x\right)^2}dx$

Ta có: $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(2+\sin x)^2}dx = 2\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{(2+\sin x)^2}dx$. *Đặt* $t = 2+\sin x$.

$\Rightarrow I = 2\int_2^3 \frac{t-2}{t^2}dt = 2\int_2^3 \left(\frac{1}{t}-\frac{2}{t^2}\right)dt = 2\left(\ln t + \frac{2}{t}\right)\Big|_2^3 = 2\ln\frac{3}{2} - \frac{2}{3}$

2. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^6 x + \cos^6 x}}dx$

- $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{1-\frac{3}{4}\sin^2 2x}}dx$. Đặt $t = 1-\frac{3}{4}\sin^2 2x \Rightarrow I = \int_1^{\frac{1}{4}}\left(-\frac{2}{3}\frac{1}{\sqrt{t}}\right)dt = \frac{4}{3}\sqrt{t}\Big|_{\frac{1}{4}}^1 = \frac{2}{3}$.

3. $I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}dx$

Đặt $t = \sqrt{3+\sin^2 x} = \sqrt{4-\cos^2 x}$. *Ta có:* $\cos^2 x = 4-t^2$ *và* $dt = \frac{\sin x\cos x}{\sqrt{3+\sin^2 x}}dx$.

$I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}.dx = \int_0^{\frac{\pi}{3}} \frac{\sin x.\cos x}{\cos^2 x\sqrt{3+\sin^2 x}}dx = \int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}} \frac{dt}{4-t^2} = \frac{1}{4}\int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}}\left(\frac{1}{t+2}-\frac{1}{t-2}\right)dt$

$= \frac{1}{4}\ln\left|\frac{t+2}{t-2}\right|\Big|_{\sqrt{3}}^{\frac{\sqrt{15}}{2}} = \frac{1}{4}\left(\ln\left|\frac{\sqrt{15}+4}{\sqrt{15}-4}\right| - \ln\left|\frac{\sqrt{3}+2}{\sqrt{3}-2}\right|\right) = \frac{1}{2}\left(\ln\left(\sqrt{15}+4\right) - \ln\left(\sqrt{3}+2\right)\right)$.

4. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x+(x+\sin x)\sin x}{\sin^3 x + \sin^2 x}dx$

$I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x}dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x}$.

+ Tính $I_1 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x}dx$. *Đặt* $\begin{cases} u = x \\ dv = \frac{dx}{\sin^2 x}\end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x\end{cases} \Rightarrow I_1 = \frac{\pi}{\sqrt{3}}$

+ Tính $I_2 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} = 4-2\sqrt{3}$

Vậy: $I = \frac{\pi}{\sqrt{3}} + 4 - 2\sqrt{3}$.

5. $I=\int\limits_0^{\frac{\pi}{2}}\frac{\sin 2x}{\sqrt{\cos^2 x+4\sin^2 x}}dx$

$I=\int\limits_0^{\frac{\pi}{2}}\frac{2\sin x\cos x}{\sqrt{3\sin^2 x+1}}dx$. Đặt $u=\sqrt{3\sin^2 x+1}\Rightarrow I=\int\limits_1^2\frac{\frac{2}{3}udu}{u}=\frac{2}{3}\int\limits_1^2 du=\frac{2}{3}$

6. $I=\int\limits_0^{\frac{\pi}{6}}\frac{\tan\left(x-\frac{\pi}{4}\right)}{\cos 2x}dx$

$I=\int\limits_0^{\frac{\pi}{6}}\frac{\tan\left(x-\frac{\pi}{4}\right)}{\cos 2x}dx=-\int\limits_0^{\frac{\pi}{6}}\frac{\tan^2 x+1}{(\tan x+1)^2}dx$. *Đặt* $t=\tan x\Rightarrow dt=\frac{1}{\cos^2 x}dx=(\tan^2 x+1)dx$

$\Rightarrow I=-\int\limits_0^{\frac{1}{\sqrt{3}}}\frac{dt}{(t+1)^2}=\left.\frac{1}{t+1}\right|_0^{\frac{1}{\sqrt{3}}}=\frac{1-\sqrt{3}}{2}$.

7. $I=2\int\limits_1^2\sqrt[6]{1-\cos^3 x}.\sin x.\cos^5 x dx$

Đặt $t=\sqrt[6]{1-\cos^3 x}\Leftrightarrow t^6=1-\cos^3 x\Rightarrow 6t^5dt=3\cos^2 x\sin xdx\Rightarrow dx=\frac{2t^5dt}{\cos^2 x\sin x}$

$\Rightarrow I=2\int\limits_0^1 t^6(1-t^6)dt=2\left.\left(\frac{t^7}{7}-\frac{t^{13}}{13}\right)\right|_0^1=\frac{12}{91}$

8. $I=\int\limits_0^{\frac{\pi}{4}}\frac{\tan xdx}{\cos x\sqrt{1+\cos^2 x}}$

Ta có: $I=\int\limits_0^{\frac{\pi}{4}}\frac{\tan xdx}{\cos^2 x\sqrt{\tan^2 x+2}}$. *Đặt* $t=\sqrt{2+\tan^2 x}\Rightarrow t^2=2+\tan^2 x\Rightarrow tdt=\frac{\tan x}{\cos^2 x}dx$

$\Rightarrow I=\int\limits_{\sqrt{2}}^{\sqrt{3}}\frac{tdt}{t}=\int\limits_{\sqrt{2}}^{\sqrt{3}}dt=\sqrt{3}-\sqrt{2}$

9. $I=\int\limits_0^{\frac{\pi}{2}}\frac{\cos 2x}{(\cos x-\sin x+3)^3}dx$

Đặt $t=\cos x-\sin x+3\Rightarrow I=\int\limits_2^4\frac{t-3}{t^3}dt=-\frac{1}{32}$.

10. $I=\int\limits_0^{\frac{\pi}{4}}\frac{\sin 4x}{\cos^2 x.\sqrt{\tan^4 x+1}}dx$

Ta có: $\mathrm{I}=\int\limits_0^{\frac{\pi}{4}}\frac{\sin 4x}{\sqrt{\sin^4 x+\cos^4 x}}dx$. *Đặt* $t=\sqrt{\sin^4 x+\cos^4 x}\;\Rightarrow \mathrm{I}=-2\int\limits_1^{\frac{\sqrt{2}}{2}}dt=2-\sqrt{2}$.

11. $I=\int\limits_0^{\frac{\pi}{4}}\frac{\sin 4x}{1+\cos^2 x}dx$

Ta có: $I=\int\limits_0^{\frac{\pi}{4}}\frac{2\sin 2x(2\cos^2 x-1)}{1+\cos^2 x}dx$. *Đặt* $t=\cos^2 x\Rightarrow I=-\int\limits_1^{\frac{1}{2}}\frac{2(2t-1)}{t+1}dt=2-6\ln\frac{1}{3}$.

12. $I=\int\limits_0^{\frac{\pi}{6}}\frac{\tan^3 x}{\cos 2x}dx$

Ta có: $I=\int\limits_0^{\frac{\pi}{6}}\frac{\tan^3 x}{\cos^2 x-\sin^2 x}dx=\int\limits_0^{\frac{\pi}{6}}\frac{\tan^3 x}{\cos^2 x(1-\tan^2 x)}dx$.

Đặt $t=\tan x\;\Rightarrow I=\int\limits_0^{\frac{\sqrt{3}}{3}}\frac{t^3}{1-t^2}dt=-\frac{1}{6}-\frac{1}{2}\ln\frac{2}{3}$.

13. $I=\int\limits_0^{\frac{\pi}{4}}\frac{\cos x-\sin x}{\sqrt{3-\sin 2x}}dx$

Đặt $u=\sin x+\cos x\;\Rightarrow I=\int\limits_1^{\sqrt{2}}\frac{du}{\sqrt{4-u^2}}$.

Đặt $u=2\sin t\;\Rightarrow I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{2\cos t\,dt}{\sqrt{4-4\sin^2 t}}=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}dt=\frac{\pi}{12}$.

14. $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sin x.\sin\left(x+\frac{\pi}{4}\right)}dx$

$I=\sqrt{2}\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sin^2 x(1+\cot x)}dx$. *Đặt* $1+\cot x=t\Rightarrow\frac{1}{\sin^2 x}dx=-dt$

$$\Rightarrow I=\sqrt{2}\int\limits_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1}\frac{t-1}{t}dt=\sqrt{2}\left(t-\ln t\right)\Big|_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1}=\sqrt{2}\left(\frac{2}{\sqrt{3}}-\ln\sqrt{3}\right)$$

15. $I=\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 x.\cos^4 x}$

Ta có: $I=4.\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 2x.\cos^2 x}$. *Đặt* $t=\tan x\Rightarrow dx=\frac{dt}{1+t^2}$

$$\Rightarrow I=\int\limits_1^{\sqrt{3}}\frac{(1+t^2)^2dt}{t^2}=\int\limits_1^{\sqrt{3}}(\frac{1}{t^2}+2+t^2)dt=(-\frac{1}{t}+2t+\frac{t^3}{3})\Big|_1^{\sqrt{3}}=\frac{8\sqrt{3}-4}{3}$$

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HT 5.Tính các tích phân sau:

1. $I=\int\frac{\sin 2x dx}{3+4\sin x-\cos 2x}$	**11.** $I=\int\limits_0^{\frac{\pi}{6}}\frac{1}{\sin x+\sqrt{3}\cos x}dx$
2. $I=\int\frac{dx}{\sin^3 x.\cos^5 x}$	**12.** $I=\int\limits_0^{\frac{\pi}{2}}\sqrt{1-\sqrt{3}\sin 2x+2\cos^2 x}dx$
3. $I=\int\frac{dx}{\sin x.\cos^3 x}$	**13.** $I=\int\limits_0^{\frac{\pi}{4}}\frac{\sin x}{5\sin x.\cos^2 x+2\cos x}dx$
4. $I=\int\limits_0^{\frac{\pi}{2}}\frac{\sin 2x.\cos x}{1+\cos x}dx$	**14.** $I=\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin^2 x dx}{\cos^4 x(\tan^2 x-2\tan x+5)}$
5. $I=\int\limits_0^{\frac{\pi}{3}}\sin^2 x\tan x dx$	**15.** $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\sin^2 x}{\sin 3x}dx$
6. $I=\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 x(2-\sqrt{1+\cos 2x})dx$	**16.** $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\sin x-\cos x}{\sqrt{1+\sin 2x}}dx$
7. $I=\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 x.\cos^4 x}$	**17.** $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sqrt[4]{\sin^3 x.\cos^5 x}}$
8. $I=\int\limits_0^{\frac{\pi}{6}}\frac{\sin x}{\cos 2x}dx$	

9. $I=\int\limits_0^{\frac{\pi}{2}}\dfrac{\sin x}{\left(\sin x+\sqrt{3}\cos x\right)^3}dx$ **10.** $I=\int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{4}}\dfrac{\sin x\sqrt{1-\cos^2 x}}{\cos^2 x}dx$	**18.** $I=\int\limits_0^{\pi}x\left(\dfrac{\cos^3 x+\cos x+\sin x}{1+\cos^2 x}\right)dx$ **19.** $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\dfrac{\cos x}{\sin x\sqrt{3+\cos^2 x}}dx$

Bài giải

1. $I=\int\dfrac{\sin 2xdx}{3+4\sin x-\cos 2x}$

Ta có: $I=\int\dfrac{2\sin x\cos x}{2\sin^2 x+4\sin x+2}dx$. *Đặt* $t=\sin x\Rightarrow I=\ln\left|\sin x+1\right|+\dfrac{1}{\sin x+1}+C$

2. $I=\int\dfrac{dx}{\sin^3 x.\cos^5 x}$

$$I=\int\frac{dx}{\sin^3 x.\cos^3 x.\cos^2 x}=8\int\frac{dx}{\sin^3 2x.\cos^2 x}$$

Đặt $t=\tan x$. $I=\int\left(t^3+3t+\dfrac{3}{t}+t^{-3}\right)dt=\dfrac{1}{4}\tan^4 x+\dfrac{3}{2}\tan^2 x+3\ln\left|\tan x\right|-\dfrac{1}{2\tan^2 x}+C$

Chú ý: $\sin 2x=\dfrac{2t}{1+t^2}$.

3. $I=\int\dfrac{dx}{\sin x.\cos^3 x}$

$I=\int\dfrac{dx}{\sin x.\cos x.\cos^2 x}=2\int\dfrac{dx}{\sin 2x.\cos^2 x}$. *Đặt* $t=\tan x\Rightarrow dt=\dfrac{dx}{\cos^2 x};\ \sin 2x=\dfrac{2t}{1+t^2}$

$$\Rightarrow I=2\int\frac{dt}{\frac{2t}{1+t^2}}=\int\frac{t^2+1}{t}dt=\int\left(t+\frac{1}{t}\right)dt=\frac{t^2}{2}+\ln\left|t\right|+C=\frac{\tan^2 x}{2}+\ln\left|\tan x\right|+C$$

4. $I=\int\limits_0^{\frac{\pi}{2}}\dfrac{\sin 2x.\cos x}{1+\cos x}dx$

Ta có: $I=2\int\limits_0^{\frac{\pi}{2}}\dfrac{\sin x.\cos^2 x}{1+\cos x}dx$. *Đặt* $t=1+\cos x\Rightarrow I=2\int\limits_1^2\dfrac{(t-1)^2}{t}dt=2\ln 2-1$

5. $I=\int\limits_0^{\frac{\pi}{3}}\sin^2 x\tan xdx$

Ta có: $I=\int\limits_0^{\frac{\pi}{3}}\sin^2 x.\frac{\sin x}{\cos x}dx=\int\limits_0^{\frac{\pi}{3}}\frac{(1-\cos^2 x)\sin x}{\cos x}dx$. *Đặt* $t=\cos x$

$\Rightarrow I=-\int\limits_1^{\frac{1}{2}}\frac{1-u^2}{u}du=\ln 2-\frac{3}{8}$

6. $I=\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 x(2-\sqrt{1+\cos 2x})dx$

Ta có: $I=\int\limits_{\frac{\pi}{2}}^{\pi}2\sin^2 xdx-\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 x\sqrt{1+\cos 2x}dx=H+K$

\+ $H=\int\limits_{\frac{\pi}{2}}^{\pi}2\sin^2 xdx=\int\limits_{\frac{\pi}{2}}^{\pi}(1-\cos 2x)dx=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

\+ $K=\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 x\sqrt{2\cos^2 x}=-\sqrt{2}\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 x\cos xdx=-\sqrt{2}\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 xd(\sin x)=\frac{\sqrt{2}}{3}$

$\Rightarrow I=\frac{\pi}{2}-\frac{\sqrt{2}}{3}$

7. $I=\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 x.\cos^4 x}$

$I=4.\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 2x.\cos^2 x}$. *Đặt* $t=\tan x\Rightarrow dt=\frac{dx}{\cos^2 x}$.

$I=\int\limits_1^{\sqrt{3}}\frac{(1+t^2)^2dt}{t^2}=\int\limits_1^{\sqrt{3}}\left(\frac{1}{t^2}+2+t^2\right)dt=\left(-\frac{1}{t}+2t+\frac{t^3}{3}\right)\Bigg|_1^{\sqrt{3}}=\frac{8\sqrt{3}-4}{3}$

8. $I=\int\limits_0^{\frac{\pi}{6}}\frac{\sin x}{\cos 2x}dx$

$I=\int\limits_0^{\frac{\pi}{6}}\frac{\sin x}{\cos 2x}dx=\int\limits_0^{\frac{\pi}{6}}\frac{\sin x}{2\cos^2 x-1}dx$. *Đặt* $t=\cos x\Rightarrow dt=-\sin xdx$

Đổi cận: $x=0 \Rightarrow t=1; \quad x=\frac{\pi}{6} \Rightarrow t=\frac{\sqrt{3}}{2}$

Ta được $I=-\int_{1}^{\frac{\sqrt{3}}{2}} \frac{1}{2t^2-1}dt=\frac{1}{2\sqrt{2}}\ln\left|\frac{2t-\sqrt{2}}{2t+\sqrt{2}}\right|\Bigg|_{\frac{\sqrt{3}}{2}}^{1} = \frac{1}{2\sqrt{2}}\ln\left|\frac{3-2\sqrt{2}}{5-2\sqrt{6}}\right|$

9. $I=\int_{0}^{\frac{\pi}{2}} \frac{\sin x}{\left(\sin x+\sqrt{3}\cos x\right)^3}dx$

Ta có: $\sin x+\sqrt{3}\cos x=2\cos\left(x-\frac{\pi}{6}\right)$;

$$\sin x=\sin\left(\left(x-\frac{\pi}{6}\right)+\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\sin\left(x-\frac{\pi}{6}\right)+\frac{1}{2}\cos\left(x-\frac{\pi}{6}\right)$$

$$I=\frac{\sqrt{3}}{16}\int_{0}^{\frac{\pi}{2}}\frac{\sin\left(x-\frac{\pi}{6}\right)dx}{\cos^3\left(x-\frac{\pi}{6}\right)}+\frac{1}{16}\int_{0}^{\frac{\pi}{2}}\frac{dx}{\cos^2\left(x-\frac{\pi}{6}\right)} = \frac{\sqrt{3}}{6}$$

10. $I=\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x\sqrt{1-\cos^2 x}}{\cos^2 x}dx$

$$I=\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{\sin x}{\cos^2 x}\sqrt{1-\cos^2 x}.dx=\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{\sin x}{\cos^2 x}|\sin x|\,dx=\int_{-\frac{\pi}{3}}^{0}\frac{\sin x}{\cos^2 x}|\sin x|\,dx+\int_{-0}^{\frac{\pi}{4}}\frac{\sin x}{\cos^2 x}|\sin x|\,dx$$

$$=-\int_{-\frac{\pi}{3}}^{0}\frac{\sin^2 x}{\cos^2 x}dx+\int_{0}^{\frac{\pi}{4}}\frac{\sin^2 x}{\cos^2 x}dx=\frac{7\pi}{12}-\sqrt{3}-1.$$

11. $I=\int_{0}^{\frac{\pi}{6}} \frac{1}{\sin x+\sqrt{3}\cos x}dx$

$$I=\int_{0}^{\frac{\pi}{6}}\frac{1}{\sin x+\sqrt{3}\cos x}dx = \frac{1}{2}\int_{0}^{\frac{\pi}{6}}\frac{1}{\sin\left(x+\frac{\pi}{3}\right)}dx = \frac{1}{2}\int_{0}^{\frac{\pi}{6}}\frac{\sin\left(x+\frac{\pi}{3}\right)}{1-\cos^2\left(x+\frac{\pi}{3}\right)}dx.$$

Đặt $t=\cos\left(x+\frac{\pi}{3}\right)\Rightarrow dt=-\sin\left(x+\frac{\pi}{3}\right)dx \Rightarrow I=\frac{1}{2}\int_{0}^{\frac{1}{2}}\frac{1}{1-t^2}dt=\frac{1}{4}\ln 3$

12. $I = \int\limits_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3}\sin 2x + 2\cos^2 x}dx$

$$I = \int\limits_0^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = I = \int\limits_0^{\frac{\pi}{3}} \left|\sin x - \sqrt{3}\cos x\right| dx + \int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = 3 - \sqrt{3}$$

13. $I = \int\limits_0^{\frac{\pi}{4}} \frac{\sin x}{5\sin x.\cos^2 x + 2\cos x}dx$

Ta có: $I = \int\limits_0^{\frac{\pi}{4}} \frac{\tan x}{5\tan x + 2(1+\tan^2 x)}.\frac{1}{\cos^2 x}dx$. *Đặt* $t = \tan x$,

$$\Rightarrow I = \int\limits_0^1 \frac{t}{2t^2 + 5t + 2}dt = \frac{1}{3}\int\limits_0^1 \left(\frac{2}{t+2} - \frac{1}{2t+1}\right)dt = \frac{1}{2}\ln 3 - \frac{2}{3}\ln 2$$

14. $I = \int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^2 x dx}{\cos^4 x(\tan^2 x - 2\tan x + 5)}$

Đặt $t = \tan x \Rightarrow dx = \frac{dt}{1+t^2} \Rightarrow I = \int\limits_{-1}^1 \frac{t^2 dt}{t^2 - 2t + 5} = 2 + \ln\frac{2}{3} - 3\int\limits_{-1}^1 \frac{dt}{t^2 - 2t + 5}$

Tính $I_1 = \int\limits_{-1}^1 \frac{dt}{t^2 - 2t + 5}$. *Đặt* $\frac{t-1}{2} = \tan u \Rightarrow I_1 = \frac{1}{2}\int\limits_{-\frac{\pi}{4}}^0 du = \frac{\pi}{8}$. *Vậy* $I = 2 + \ln\frac{2}{3} - \frac{3\pi}{8}$.

15. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin 3x}dx$.

$$I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{3\sin x - 4\sin^3 x}dx = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin x}{4\cos^2 x - 1}dx$$

Đặt $t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow I = -\int\limits_{\frac{\sqrt{3}}{2}}^0 \frac{dt}{4t^2 - 1} = \frac{1}{4}\int\limits_0^{\frac{\sqrt{3}}{2}} \frac{dt}{t^2 - \frac{1}{4}} = \frac{1}{4}\ln(2 - \sqrt{3})$

16. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}}dx$

Ta có: $\sqrt{1+\sin 2x} = |\sin x + \cos x| = \sin x + \cos x$ *(vì* $x \in \left[\frac{\pi}{4};\frac{\pi}{2}\right]$*)*

$\Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sin x + \cos x}dx$. *Đặt* $t = \sin x + \cos x \Rightarrow dt = (\cos x - \sin x)dx$

$\Rightarrow I = \int_1^{\sqrt{2}} \frac{1}{t}dt = \ln|t|\Big|_1^{\sqrt{2}} = \frac{1}{2}\ln 2$

17. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sqrt[4]{\sin^3 x.\cos^5 x}}$

Ta có: $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sqrt[4]{\frac{\sin^3 x}{\cos^3 x}.\cos^8 x}}dx = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sqrt[4]{\tan^3 x}}.\frac{1}{\cos^2 x}dx$.

Đặt $t = \tan x \Rightarrow I = \int\limits_1^{\sqrt{3}} t^{-\frac{3}{4}}dt = 4\left(\sqrt[8]{3} - 1\right)$

18. $I = \int\limits_0^{\pi} x(\frac{\cos^3 x + \cos x + \sin x}{1+\cos^2 x})dx$

Ta có: $I = \int\limits_0^{\pi} x\left(\frac{\cos x(1+\cos^2 x) + \sin x}{1+\cos^2 x}\right)dx = \int\limits_0^{\pi} x.\cos x.dx + \int\limits_0^{\pi} \frac{x.\sin x}{1+\cos^2 x}dx = J + K$

+ *Tính* $J = \int\limits_0^{\pi} x.\cos x.dx$. *Đặt* $\begin{cases} u = x \\ dv = \cos x dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \sin x \end{cases} \Rightarrow J = -2$

+ *Tính* $K = \int\limits_0^{\pi} \frac{x.\sin x}{1+\cos^2 x}dx$. *Đặt* $x = \pi - t \Rightarrow dx = -dt$

$\Rightarrow K = \int\limits_0^{\pi} \frac{(\pi - t).\sin(\pi - t)}{1+\cos^2(\pi - t)}dt = \int\limits_0^{\pi} \frac{(\pi - t).\sin t}{1+\cos^2 t}dt = \int\limits_0^{\pi} \frac{(\pi - x).\sin x}{1+\cos^2 x}dx$

$\Rightarrow 2K = \int\limits_0^{\pi} \frac{(x + \pi - x).\sin x}{1+\cos^2 x}dx = \pi\int\limits_0^{\pi} \frac{\sin x.dx}{1+\cos^2 x} \Rightarrow K = \frac{\pi}{2}\int\limits_0^{\pi} \frac{\sin x.dx}{1+\cos^2 x}$

Đặt $t = \cos x \Rightarrow K = \frac{\pi}{2}\int\limits_{-1}^{1}\frac{dt}{1+t^2}$, *đặt* $t = \tan u \Rightarrow dt = (1+\tan^2 u)du$

$$\Rightarrow K = \frac{\pi}{2}\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan^2 u)du}{1+\tan^2 u} = \frac{\pi}{2}\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}}du = \frac{\pi}{2}.u\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi^2}{4}$$

Vậy $I = \frac{\pi^2}{4} - 2$

19. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos x}{\sin x\sqrt{3+\cos^2 x}}dx$

Ta có: $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^2 x\sqrt{3+\cos^2 x}}dx$. *Đặt* $t = \sqrt{3+\cos^2 x}$

$$\Rightarrow I = \int\limits_{\sqrt{3}}^{\frac{\sqrt{15}}{2}}\frac{dt}{4-t^2} = \frac{1}{2}\left(\ln(\sqrt{15}+4) - \ln(\sqrt{3}+2)\right)$$

HT 6.Tính các tích phân sau:

1. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x\cdot\sqrt{\sin^2 x + \frac{1}{2}}.dx$

2. $I = \int\limits_{0}^{\frac{\pi}{2}}\frac{3\sin x + 4\cos x}{3\sin^2 x + 4\cos^2 x}dx$

3. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\tan x}{\cos x\sqrt{1+\cos^2 x}}dx$

4. $I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\sin\left(x+\frac{\pi}{4}\right)}{2\sin x\cos x - 3}dx$

Bài giải

1. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x\cdot\sqrt{\sin^2 x + \frac{1}{2}}.dx$

• *Đặt* $\cos x = \sqrt{\frac{3}{2}}\sin t, \left(0 \le t \le \frac{\pi}{2}\right) \Rightarrow I = \frac{3}{2}\int\limits_{0}^{\frac{\pi}{4}}\cos^2 t dt = \frac{3}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right).$

2. $I=\int\limits_0^{\frac{\pi}{2}}\frac{3\sin x+4\cos x}{3\sin^2 x+4\cos^2 x}dx$

• $I=\int\limits_0^{\frac{\pi}{2}}\frac{3\sin x+4\cos x}{3+\cos^2 x}dx=\int\limits_0^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx+\int\limits_0^{\frac{\pi}{2}}\frac{4\cos x}{3+\cos^2 x}dx=\int\limits_0^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx+\int\limits_0^{\frac{\pi}{2}}\frac{4\cos x}{4-\sin^2 x}dx$

+ *Tính* $I_1=\int\limits_0^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx$. *Đặt* $t=\cos x\Rightarrow dt=-\sin xdx\ \Rightarrow I_1=\int\limits_0^{1}\frac{3dt}{3+t^2}$

Đặt $t=\sqrt{3}\tan u\Rightarrow dt=\sqrt{3}(1+\tan^2 u)du\ \Rightarrow I_1=\int\limits_0^{\frac{\pi}{6}}\frac{3\sqrt{3}(1+\tan^2 u)du}{3(1+\tan^2 u)}=\frac{\pi\sqrt{3}}{6}$

+ *Tính* $I_2=\int\limits_0^{\frac{\pi}{2}}\frac{4\cos x}{4-\sin^2 x}dx$. *Đặt* $t_1=\sin x\Rightarrow dt_1=\cos xdx\ \ I_2=\int\limits_0^{1}\frac{4dt_1}{4-t_1^2}dt_1=\ln 3$

Vậy: $I=\frac{\pi\sqrt{3}}{6}+\ln 3$

3. $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\tan x}{\cos x\sqrt{1+\cos^2 x}}dx$

•*Ta có:* $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\tan x}{\cos^2 x\sqrt{\frac{1}{\cos^2 x}+1}}dx=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\tan x}{\cos^2 x\sqrt{\tan^2 x+2}}dx$

Đặt $u=\tan x\Rightarrow du=\frac{1}{\cos^2 x}dx\ \Rightarrow I=\int\limits_{\frac{1}{\sqrt{3}}}^{1}\frac{u}{\sqrt{u^2+2}}dx$. *Đặt* $t=\sqrt{u^2+2}\Rightarrow dt=\frac{u}{\sqrt{u^2+2}}du$.

$\Rightarrow I=\int\limits_{\sqrt{\frac{7}{3}}}^{\sqrt{3}}dt=t\Big|_{\sqrt{\frac{7}{3}}}^{\sqrt{3}}=\sqrt{3}-\frac{\sqrt{7}}{\sqrt{3}}=\frac{3-\sqrt{7}}{\sqrt{3}}.$

4. $I=\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\sin\left(x+\frac{\pi}{4}\right)}{2\sin x\cos x-3}dx$

$\bullet$*Ta có:* $I = -\frac{1}{\sqrt{2}}\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\left(\sin x - \cos x\right)^2 + 2}dx$. *Đặt* $t = \sin x - \cos x \Rightarrow I = -\frac{1}{\sqrt{2}}\int\limits_0^1 \frac{1}{t^2+2}dt$

Đặt $t = \sqrt{2}\tan u \Rightarrow I = -\frac{1}{\sqrt{2}}\int\limits_0^{\arctan\frac{1}{\sqrt{2}}} \frac{\sqrt{2}(1+\tan^2 u)}{2\tan^2 u + 2}du = -\frac{1}{2}\arctan\frac{1}{\sqrt{2}}$

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HT 7.Tính các tích phân sau:

1. $I = \int\limits_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{x\sin x}{\cos^2 x}dx$ 2. $I = \int\limits_0^{\frac{\pi}{2}} \left(\frac{1+\sin x}{1+\cos x}\right).e^x dx$ 3. $I = \int\limits_0^{\frac{\pi}{4}} \frac{x\cos 2x}{\left(1+\sin 2x\right)^2}dx$

Bài giải

1. $I = \int\limits_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{x\sin x}{\cos^2 x}dx$.

$\bullet$*Sử dụng công thức tích phân từng phần ta có:*

$$I = \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} xd\left(\frac{1}{\cos x}\right) = \frac{x}{\cos x}\Bigg|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} - \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{\cos x} = \frac{4\pi}{3} - J,\ \textit{với}\ J = \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{\cos x}$$

Để tính J ta đặt $t = \sin x$. *Khi đó* $J = \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{\cos x} = \int\limits_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{dt}{1-t^2} = -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Bigg|_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} = -\ln\frac{2-\sqrt{3}}{2+\sqrt{3}}$

Vậy $I = \frac{4\pi}{3} - \ln\frac{2-\sqrt{3}}{2+\sqrt{3}}$.

2. $I = \int\limits_0^{\frac{\pi}{2}} \left(\frac{1+\sin x}{1+\cos x}\right).e^x dx$

Ta có: $\frac{1+\sin x}{1+\cos x} = \frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{1}{2\cos^2\frac{x}{2}} + \tan\frac{x}{2}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{e^x dx}{2\cos^2 \frac{x}{2}} + \int_0^{\frac{\pi}{2}} e^x \tan\frac{x}{2} dx = e^{\frac{\pi}{2}}$$

3. $I = \int_0^{\frac{\pi}{4}} \frac{x\cos 2x}{\left(1+\sin 2x\right)^2} dx$

Đặt $\begin{cases} u = x \\ dv = \dfrac{\cos 2x}{(1+\sin 2x)^2} dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\dfrac{1}{1+\sin 2x} \end{cases}$

$$\Rightarrow I = x.\left(-\frac{1}{2}.\frac{1}{1+\sin 2x}\right)\Bigg|_0^{\frac{\pi}{4}} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin 2x} dx = -\frac{\pi}{16} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{2}}.\frac{1}{\cos^2\left(x-\frac{\pi}{4}\right)} dx$$

$$= -\frac{\pi}{16} + \frac{1}{2}.\frac{1}{\sqrt{2}}\tan\left(x-\frac{\pi}{4}\right)\Bigg|_0^{\frac{\pi}{4}} = -\frac{\pi}{16} + \frac{1}{2}.\frac{\sqrt{2}}{2}\left(0+1\right) = \frac{\sqrt{2}}{4} - \frac{\pi}{16}$$

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PHẦN V TÍCH PHÂN HÀM MŨ VÀ LOGARIT

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HT 1.Tính các tích phân sau:

1. $I = \int \frac{e^{2x}}{1+\sqrt{e^x}}dx$	**10.** $I = \int\limits_{\ln 2}^{\ln 3} \frac{e^{2x}dx}{e^x - 1 + \sqrt{e^x - 2}}$
2. $I = \int \frac{(x^2+x)e^x}{x+e^{-x}}dx$	**11.** $I = \int\limits_{0}^{\ln 3} \frac{2e^{3x} - e^{2x}}{e^x\sqrt{4e^x - 3} + 1}dx$
3. $I = \int \frac{dx}{\sqrt{e^{2x}+9}}$	**12.** $I = \int\limits_{\ln\frac{8}{3}}^{\ln\frac{16}{3}} \sqrt{3e^x - 4}dx$
4. $I = \int \frac{\ln(1+x^2)^x + 2011x}{\ln\left[(ex^2+e)^{x^2+1}\right]}dx$	**13.** $I = \int\limits_{0}^{\ln 3} \frac{e^x}{\sqrt{(e^x+1)^3}}dx$
5. $J = \int\limits_{1}^{e} \frac{xe^x+1}{x(e^x+\ln x)}dx$	**14.** $I = \int\limits_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}}dx$
6. $I = \int\limits_{0}^{\ln 2} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^x+1}dx$	**15.** $I = \int\limits_{0}^{\ln 2} \sqrt{e^x - 1}dx$
7. $I = \int\limits_{0}^{3\ln 2} \frac{dx}{\left(\sqrt[3]{e^x}+2\right)^2}$	**16.** $I = \int\limits_{1}^{2} \frac{2^x - 2^{-x}}{4^x + 4^{-x} - 2}dx$
8. $I = \int\limits_{0}^{\ln 2} \sqrt[3]{e^x - 1}\, dx$	**17.** $I = \int\limits_{0}^{1} \frac{6^x dx}{9^x + 3.6^x + 2.4^x}$
9. $I = \int\limits_{3\ln 2}^{\ln 15} \frac{\left(e^{2x} - 24e^x\right)dx}{e^x\sqrt{e^x+1} + 5e^x - 3\sqrt{e^x+1} - 15}$	

Bài giải

1. $I = \int \frac{e^{2x}}{1+\sqrt{e^x}}dx$

Đặt $t = \sqrt{e^x} \Rightarrow e^x = t^2 \Rightarrow e^x dx = 2tdt$.

$$\Rightarrow I = 2\int \frac{t^3}{1+t}dt = \frac{2}{3}t^3 - t^2 + 2t - 2\ln|t+1| + C = \frac{2}{3}e^x\sqrt{e^x} - e^x + 2\sqrt{e^x} - 2\ln\left|\sqrt{e^x}+1\right| + C$$

2. $I = \int \frac{(x^2+x)e^x}{x+e^{-x}}dx$

$I = \int \frac{(x^2+x)e^x}{x+e^{-x}}dx = \int \frac{xe^x.(x+1)e^x}{xe^x+1}dx$. **Đặt** $t = x.e^x + 1 \Rightarrow I = xe^x + 1 - \ln|xe^x+1| + C$.

3. $I = \int \frac{dx}{\sqrt{e^{2x}+9}}$

Đặt $t = \sqrt{e^{2x}+9} \Rightarrow I = \int \frac{dt}{t^2-9} = \frac{1}{6}\ln\left|\frac{t-3}{t+3}\right| + C = \frac{1}{6}\ln\left|\frac{\sqrt{e^{2x}+9}-3}{\sqrt{e^{2x}+9}+3}\right| + C$

4. $I = \int \frac{\ln(1+x^2)^x + 2011x}{\ln\left[(ex^2+e)^{x^2+1}\right]}dx$

Ta có: $I = \int \frac{x\left[\ln(x^2+1)+2011\right]}{(x^2+1)\left[\ln(x^2+1)+1\right]}dx$. *Đặt* $t = \ln(x^2+1)+1$

$$\Rightarrow I = \frac{1}{2}\int \frac{t+2010}{t}dt = \frac{1}{2}t + 1005\ln\left|t\right| + C = \frac{1}{2}\ln(x^2+1) + \frac{1}{2} + 1005\ln(\ln(x^2+1)+1) + C$$

5. $J = \int\limits_1^e \frac{xe^x+1}{x(e^x+\ln x)}dx$ $\quad J = \int\limits_1^e \frac{d(e^x+\ln x)}{e^x+\ln x} = \ln\left|e^x+\ln x\right|\Big|_1^e = \ln\frac{e^e+1}{e}$

6. $I = \int\limits_0^{\ln 2} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^x+1}dx$

$$I = \int\limits_0^{\ln 2} \frac{3e^{3x}+2e^{2x}-e^x-(e^{3x}+e^{2x}-e^x+1)}{e^{3x}+e^{2x}-e^x+1}dx = \int\limits_0^{\ln 2}\left(\frac{3e^{3x}+2e^{2x}-e^x}{e^{3x}+e^{2x}-e^x+1}-1\right)dx$$

$$= \ln(e^{3x}+e^{2x}-e^x+1)\Big|_0^{\ln 2} - x\Big|_0^{\ln 2} = \ln 11 - \ln 4 = \ln\frac{14}{4}$$

7. $I = \int\limits_0^{3\ln 2} \frac{dx}{\left(\sqrt[3]{e^x}+2\right)^2}$

$$I = \int\limits_0^{3\ln 2} \frac{e^{\frac{x}{3}}dx}{e^{\frac{x}{3}}\left(\sqrt[3]{e^x}+2\right)^2}$$. *Đặt* $t = e^{\frac{x}{3}} \Rightarrow dt = \frac{1}{3}e^{\frac{x}{3}}dx \Rightarrow I = \frac{3}{4}\left(\ln\frac{3}{2}-\frac{1}{6}\right)$

8. $I = \int\limits_0^{\ln 2} \sqrt[3]{e^x-1}\,dx$

Đặt $\sqrt[3]{e^x-1} = t \Rightarrow dx = \frac{3t^2dt}{t^3+1} \Rightarrow I = 3\int\limits_0^1\left(1-\frac{1}{t^3+1}\right)dt = 3 - 3\int\limits_0^1 \frac{dt}{t^3+1}$.

Tính $I_1 = 3\int\limits_0^1 \frac{dt}{t^3+1} = \int\limits_0^1\left(\frac{1}{t+1}+\frac{2-t}{t^2-t+1}\right)dt = \frac{\pi}{\sqrt{3}} + \ln 2$

Vậy: $I = 3 - \ln 2 - \frac{\pi}{\sqrt{3}}$

9. $I = \int\limits_{3\ln 2}^{\ln 15} \frac{\left(e^{2x}-24e^x\right)dx}{e^x\sqrt{e^x+1}+5e^x-3\sqrt{e^x+1}-15}$

Đặt $t = \sqrt{e^x+1} \Rightarrow t^2-1 = e^x \Rightarrow e^xdx = 2tdt$.

$$I=\int_3^4\frac{(2t^2-10t)dt}{t^2-4}=\int_3^4\left(2-\frac{3}{t-2}-\frac{7}{t+2}\right)dt=\left(2t-3\ln\left|t-2\right|-7\ln\left|t+2\right|\right)\Big|_3^4=2-3\ln2-7\ln6+7\ln5$$

10. $I=\int_{\ln 2}^{\ln 3}\frac{e^{2x}dx}{e^x-1+\sqrt{e^x-2}}$

Đặt t = $\sqrt{e^x-2}\ \Rightarrow\ e^{2x}dx=2tdt$

$$\Rightarrow I=2\int_0^1\frac{(t^2+2)tdt}{t^2+t+1}=2\int_0^1\left(t-1+\frac{2t+1}{t^2+t+1}\right)dt=2\int_0^1(t-1)dt+2\int_0^1\frac{d(t^2+t+1)}{t^2+t+1}$$

$$=(t^2-2t)\Big|_0^1+2\ln(t^2+t+1)\Big|_0^1=2\ln3-1.$$

11. $I=\int_0^{\ln 3}\frac{2e^{3x}-e^{2x}}{e^x\sqrt{4e^x-3}+1}dx$

Đặt $t=\sqrt{4e^{3x}-3e^{2x}}\Rightarrow t^2=4e^{3x}-3e^{2x}\Rightarrow 2tdt=(12e^{3x}-6e^{2x})dx\Rightarrow(2e^{3x}-e^{2x})dx=\frac{tdt}{3}$

$$\Rightarrow I=\frac{1}{3}\int_1^9\frac{tdt}{t+1}=\frac{1}{3}\int_1^9\left(1-\frac{1}{t+1}\right)dt=\frac{1}{3}\left(t-\ln\left|t+1\right|\right)\Big|_1^9=\frac{8-\ln5}{3}.$$

12. $I=\int_{\ln\frac{8}{3}}^{\ln\frac{16}{3}}\sqrt{3e^x-4}dx$

Đặt: $t=\sqrt{3e^x-4}\Rightarrow e^x=\frac{t^2+4}{3}\Rightarrow dx=\frac{2tdt}{t^2+4}$

$$\Rightarrow I=\int_2^{2\sqrt3}\frac{2t^2}{t^2+4}dt=2\int_2^{2\sqrt3}dt-8\int_2^{2\sqrt3}\frac{dt}{t^2+4}=4\left(\sqrt3-1\right)-8I_1\text{, với } I_1=\int_2^{2\sqrt3}\frac{dt}{t^2+4}$$

Tính $I_1=\int_2^{2\sqrt3}\frac{dt}{t^2+4}$. *Đặt:* $t=2\tan u,\ u\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right)\Rightarrow dt=2(1+\tan^2u)du$

$$\Rightarrow I_1=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{2}du=\frac{1}{2}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\pi}{24}.\quad \textit{Vậy: } I=4(\sqrt3-1)-\frac{\pi}{3}$$

13. $I=\int_0^{\ln 3}\frac{e^x}{\sqrt{(e^x+1)^3}}dx$

Đặt $t=\sqrt{e^x+1}\Leftrightarrow t^2=e^x+1\Leftrightarrow 2tdt=e^xdx\Rightarrow dx=\dfrac{2tdt}{e^x}$ $\Rightarrow I=2\int\limits_{\sqrt{2}}^{2}\dfrac{tdt}{t^3}=\sqrt{2}-1$

14. $I=\int\limits_{\ln 2}^{\ln 5}\dfrac{e^{2x}}{\sqrt{e^x-1}}dx$

Đặt $t=\sqrt{e^x-1}\Leftrightarrow t^2=e^x-1\Rightarrow dx=\dfrac{2tdt}{e^x}\Rightarrow I=2\int\limits_{1}^{2}(t^2+1)d=2\left(\dfrac{t^3}{3}+t\right)\Big|_1^2=\dfrac{20}{3}$

15. $I=\int\limits_{0}^{\ln 2}\sqrt{e^x-1}dx$

Đặt $t=\sqrt{e^x-1}\Rightarrow t^2=e^x-1\Rightarrow 2tdt=e^xdx\Rightarrow dx=\dfrac{2td}{e^x}=\dfrac{2td}{t^2+1}$

$$\Rightarrow I=\int\limits_{0}^{1}\dfrac{2t^2}{t^2+1}dt=2\int\limits_{0}^{1}\left(1-\dfrac{1}{t^2+1}\right)dt=\dfrac{4-\pi}{2}$$

16. $I=\int\limits_{1}^{2}\dfrac{2^x-2^{-x}}{4^x+4^{-x}-2}dx$

Đặt $t=2^x+2^{-x}\Rightarrow 4^x+4^{-x}-2=(2^x+2^{-x})^2-4\Rightarrow I=\dfrac{1}{4\ln 2}\ln\dfrac{81}{25}$

17. $I=\int\limits_{0}^{1}\dfrac{6^x dx}{9^x+3.6^x+2.4^x}$

Ta có: $I=\int\limits_{0}^{1}\dfrac{\left(\dfrac{3}{2}\right)^x dx}{\left(\dfrac{3}{2}\right)^{2x}+3\left(\dfrac{3}{2}\right)^x+2}$. *Đặt* $t=\left(\dfrac{3}{2}\right)^x$. $I=\dfrac{1}{\ln 3-\ln 2}\int\limits_{1}^{\frac{3}{2}}\dfrac{dt}{t^2+3t+2}=\dfrac{\ln 15-\ln 14}{\ln 3-\ln 2}$

HT 2.Tính các tích phân sau:

1. $I=\int\limits_{1}^{e}\left(\dfrac{\ln x}{x\sqrt{1+\ln x}}+3x^2\ln x\right)dx$	9. $I=\int\limits_{2}^{5}\dfrac{\ln(\sqrt{x-1}+1)}{x-1+\sqrt{x-1}}dx$
2. $I=\int\limits_{1}^{e}\dfrac{\ln x\sqrt[3]{2+\ln^2 x}}{x}dx$	10. $I=\int\limits_{1}^{e^3}\dfrac{\ln^3 x}{x\sqrt{1+\ln x}}dx$
3. $I=\int\limits_{e}^{e^2}\dfrac{dx}{x\ln x.\ln ex}$	11. $I=\int\limits_{1}^{\sqrt{e}}\dfrac{3-2\ln x}{x\sqrt{1+2\ln x}}dx$
4. $I=\int\limits_{\ln 4}^{\ln 6}\dfrac{e^{2x}}{e^x+6e^{-x}-5}dx$	12. $I=\int\limits_{1}^{e}\dfrac{\ln x\sqrt[3]{2+\ln^2 x}}{x}dx$

5. $I=\int\limits_1^e \frac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}}dx$ 6. $I=\int\limits_1^e \frac{x+(x-2)\ln x}{x(1+\ln x)}dx$ 7. $I=\int\limits_{e^2}^{e^3} \frac{2x\ln^2 x - x\ln x^2+3}{x(1-\ln x)}dx$ 8. $I=\int\limits_1^{e^2} \frac{\sqrt{\ln^2 x-\ln x^2+1}}{x^2}dx$	13. $I=\int\limits_1^e \frac{xe^x+1}{x(e^x+\ln x)}dx$

Bài giải

1. $I=\int\limits_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}}+3x^2\ln x\right)dx$

$I=\int\limits_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx+3\int\limits_1^e x^2\ln x dx$ = $\frac{2(2-\sqrt{2})}{3}$ + $\frac{2e^3+1}{3}$ = $\frac{5-2\sqrt{2}+2e^3}{3}$

2. $I=\int\limits_1^e \frac{\ln x\sqrt[3]{2+\ln^2 x}}{x}dx$

Đặt $t=2+\ln^2 x \Rightarrow dt=\frac{2\ln x}{x}dx \Rightarrow I=\frac{1}{2}\int\limits_2^3 \sqrt[3]{t}dt = \frac{3}{8}\left(\sqrt[3]{3^4}-\sqrt[3]{2^4}\right)$

3. $I=\int\limits_e^{e^2} \frac{dx}{x\ln x.\ln \text{ex}}$

$I=\int\limits_e^{e^2} \frac{dx}{x\ln x(1+\ln x)}=\int\limits_e^{e^2} \frac{d(\ln x)}{\ln x(1+\ln x)}$ = $\int\limits_e^{e^2}\left(\frac{1}{\ln x}-\frac{1}{1+\ln x}\right)d(\ln x)$ = 2ln2 – ln3

4. $I=\int\limits_{\ln 4}^{\ln 6} \frac{e^{2\text{x}}}{e^x+6e^{-x}-5}dx$ •*Đặt* $t=e^x$. $I=2+9\ln 3-4\ln 2$

5. $I=\int\limits_1^e \frac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}}dx$

$I=\int\limits_1^e \frac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}}dx=\int\limits_1^e \frac{\left(\frac{\ln x}{\ln 2}\right)^3}{x\sqrt{1+3\ln^2 x}}dx=\frac{1}{\ln^3 2}\int\limits_1^e \frac{\ln^2 x.}{\sqrt{1+3\ln^2 x}}.\frac{\ln x dx}{x}$

Đặt $\sqrt{1+3\ln^2 x}=t \Rightarrow \ln^2 x=\frac{1}{3}(t^2-1) \Rightarrow \ln x.\frac{dx}{x}=\frac{1}{3}tdt$.

Suy ra $I=\frac{1}{9\ln^3 2}\left(\frac{1}{3}t^3-t\right)\Bigg|_1^2=\frac{4}{27\ln^3 2}$.

6. $I=\int\limits_1^e \frac{x+(x-2)\ln x}{x(1+\ln x)}dx$

$\int\limits_1^e dx - 2\int\limits_1^e \frac{\ln x}{x(1+\ln x)}dx = e-1-2\int\limits_1^e \frac{\ln x}{x(1+\ln x)}dx$

Tính $J=\int\limits_1^e \frac{\ln x}{x(1+\ln x)}dx$. *Đặt* $t=1+\ln x \Rightarrow J=\int\limits_1^2 \frac{t-1}{t}dt=1-\ln 2$.

Vậy: $I=e-3+2\ln 2$.

7. $I=\int\limits_{e^2}^{e^3} \frac{2x\ln^2 x - x\ln x^2+3}{x(1-\ln x)}dx$

$I=3\int\limits_{e^2}^{e^3} \frac{1}{x(1-\ln x)}dx-2\int\limits_{e^2}^{e^3} \ln x dx = -3\ln 2-4e^3+2e^2$.

8. $I=\int\limits_1^{e^2} \frac{\sqrt{\ln^2 x-\ln x^2+1}}{x^2}dx$

Đặt: $t=\ln x \Rightarrow dt=\frac{dx}{x} \Rightarrow I=\int_0^2 \frac{\sqrt{t^2-2t+1}}{e^t}dt=\int_0^2 \frac{|t-1|}{e^t}dt=-\int_0^1 \frac{t-1}{e^t}dt+\int_1^2 \frac{t-1}{e^t}dt=I_1+I_2$

\+ $I_1=-\left(\int_0^1 \frac{tdt}{e^t}-\int_0^1 \frac{dt}{e^t}\right)=-\left(-te^{-t}\Big|_0^1+\int_0^1 \frac{dt}{e^t}-\int_0^1 \frac{dt}{e^t}\right)=\frac{1}{e}$

\+ $I_2=\int_1^2 \frac{tdt}{e^t}-\int_1^2 \frac{dt}{e^t}=-te^{-t}\Big|_1^2+\int_1^2 \frac{dt}{e^t}-\int_1^2 \frac{dt}{e^t}=-te^{-t}\Big|_1^2=\frac{1}{e}-\frac{2}{e^2}$

Vậy: $I=\frac{2(e-1)}{e^2}$

9. $I=\int\limits_2^5 \frac{\ln(\sqrt{x-1}+1)}{x-1+\sqrt{x-1}}dx$

Đặt $t=\ln\left(\sqrt{x-1}+1\right) \Rightarrow 2dt=\frac{dx}{x-1+\sqrt{x-1}} \Rightarrow I=2\int\limits_{\ln 2}^{\ln 3} dt=\ln^2 3-\ln^2 2$.

10. $I=\int\limits_1^{e^3} \frac{\ln^3 x}{x\sqrt{1+\ln x}}dx$

Đặt $t=\sqrt{1+\ln x} \Rightarrow 1+\ln x=t^2 \Rightarrow \frac{dx}{x}=2tdt$ *và* $\ln^3 x=(t^2-1)^3$

$\Rightarrow I=\int\limits_1^2 \frac{(t^2-1)^3}{t}dt = \int\limits_1^2 \frac{t^6-3t^4+3t^2-1}{t}dt=\int\limits_1^2 (t^5-3t^3+3t-\frac{1}{t})dt=\frac{15}{4}-\ln 2$

11. $I=\int\limits_1^{\sqrt{e}}\frac{3-2\ln x}{x\sqrt{1+2\ln x}}dx$

Đặt $t=\sqrt{1+2\ln x}\Rightarrow I=\int\limits_1^{\sqrt{e}}(2-t^2)dt=\frac{4\sqrt{2}-5}{3}$

12. $I=\int\limits_1^{e}\frac{\ln x\sqrt[3]{2+\ln^2 x}}{x}dx$

Đặt $t=2+\ln^2 x\Rightarrow I=\frac{3}{8}\left[\sqrt[3]{3^4}-\sqrt[3]{2^4}\right]$

13. $I=\int\limits_1^{e}\frac{xe^x+1}{x(e^x+\ln x)}dx$

Đặt $t=e^x+\ln x\Rightarrow I=\ln\frac{e^e+1}{e}.$

HT 3.Tính các tích phân sau:

1. $I=\int\limits_0^{\frac{\pi}{2}}e^{\sin x}.\sin 2xdx$	*8.* $I=\int\limits_0^{\frac{1}{2}}x\ln\left(\frac{1+x}{1-x}\right)dx$
2. $I=\int\limits_0^{1}x\ln(x^2+x+1)dx$	*9.* $I=\int\limits_1^{2}x^2.\ln\left(x+\frac{1}{x}\right)dx$
3. $I=\int\limits_3^{8}\frac{\ln x}{\sqrt{x+1}}dx$	*10.* $I=\int\limits_0^{1}x^2.\ln(1+x^2)dx$
4. $I=\int\limits_1^{e}\frac{x^2+x\ln x+1}{x}e^xdx$	*11.* $I=\int\limits_1^{3}\frac{\ln x}{(x+1)^2}dx$
5. $I=\int\limits_1^{e}\left(\frac{\ln x}{x\sqrt{1+\ln x}}+\ln^2 x\right)dx$	*12.* $I=\int\limits_1^{e}\frac{\ln^2 x+e^x(e^x+\ln^2 x)}{1+e^x}.dx$
6. $I=\int\limits_1^{2}\frac{\ln(x^2+1)}{x^3}dx$	*13.* $I=\int\limits_{\frac{1}{2}}^{2}(x+1-\frac{1}{x})e^{x+\frac{1}{x}}dx$
7. $I=\int\limits_1^{2}\frac{\ln(x+1)}{x^2}dx$	*14.* $I=\int\limits_0^{4}\ln(\sqrt{x^2+9}-x)dx$

Bài giải

1. $I=\int\limits_0^{\frac{\pi}{2}}e^{\sin x}.\sin 2xdx$

$I = 2\int\limits_0^{\frac{\pi}{2}} e^{\sin x}.\sin x\cos xdx$. *Đặt* $\begin{cases} u = \sin x \\ dv = e^{\sin x}\cos xdx \end{cases} \Rightarrow \begin{cases} du = \cos xdx \\ v = e^{\sin x} \end{cases}$

$$\Rightarrow I = 2\sin xe^{\sin x}\Big|_0^{\frac{\pi}{2}} - \int\limits_0^{\frac{\pi}{2}} e^{\sin x}.\cos xdx = 2e - 2e^{\sin x}\Big|_0^{\frac{\pi}{2}} = 2$$

2. $I = \int\limits_0^1 x\ln(x^2 + x + 1)dx$

Đặt $\begin{cases} u = \ln(x^2 + x + 1) \\ dv = xdx \end{cases} \Rightarrow \begin{cases} du = \dfrac{2x+1}{x^2 + x + 1}dx \\ v = \dfrac{x^2}{2} \end{cases}$

$$I = \frac{x^2}{2}\ln(x^2 + x + 1)\Big|_0^1 - \frac{1}{2}\int\limits_0^1 \frac{2x^3 + x^2}{x^2 + x + 1}dx \quad = \frac{1}{2}\ln 3 - \frac{1}{2}\int\limits_0^1 (2x - 1)dx + \frac{1}{4}\int\limits_0^1 \frac{2x+1}{x^2 + x + 1}dx - \frac{3}{4}\int\limits_0^1 \frac{dx}{x^2 + x + 1}$$

$$= \frac{3}{4}\ln 3 - \frac{\sqrt{3}\pi}{12}$$

3. $I = \int\limits_3^8 \frac{\ln x}{\sqrt{x+1}}dx$

Đặt $\begin{cases} u = \ln x \\ dv = \dfrac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \dfrac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = \left(2\sqrt{x+1}.\ln x\right)\Big|_3^8 - 2\int\limits_3^8 \frac{\sqrt{x+1}}{x}dx = 6\ln 8 - 4\ln 3 - 2J$

+ Tính $J = \int\limits_3^8 \frac{\sqrt{x+1}}{x}dx$. *Đặt* $t = \sqrt{x+1} \Rightarrow J = \int\limits_2^3 \frac{t}{t^2 - 1}.2tdt = 2\int\limits_2^3 \frac{t^2}{t^2 - 1}dt = \int\limits_2^3 \left(2 + \frac{1}{t-1} - \frac{1}{t+1}\right)dt$

$$= \left(2t + \ln\left|\frac{t-1}{t+1}\right|\right)\Big|_3^8 = 2 + \ln 3 - \ln 2$$

Từ đó $I = 20\ln 2 - 6\ln 3 - 4$.

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4. $\mathrm{I} = \int\limits_1^e \frac{x^2 + x\ln x + 1}{x}e^x dx$

$I = \int\limits_1^e xe^x dx + \int\limits_1^e \ln xe^x dx + \int\limits_1^e \frac{e^x}{x}dx$. *+ Tính* $I_1 = \int\limits_1^e xe^x dx = xe^x\Big|_1^e - \int\limits_1^e e^x dx = e^e(e - 1)$

+Tính $I_2 = \int\limits_1^e e^x\ln xdx = e^x\ln x\Big|_1^e - \int\limits_1^e \frac{e^x}{x}dx = e^e - \int\limits_1^e \frac{e^x}{x}dx$.

Vậy: $I = I_1 + I_2 + \int\limits_1^e \frac{e^x}{x}dx = e^{e+1}$.

5. $I = \int\limits_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + \ln^2 x\right)dx$

Tính $I_1 = \int\limits_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx$. *Đặt* $t = \sqrt{1+\ln x} \Rightarrow I_1 = \frac{4}{3} - \frac{2\sqrt{2}}{3}$.

+ Tính $I_2 = \int\limits_1^e \ln^2 x dx$. *Lấy tích phân từng phần 2 lần được* $I_2 = e - 2$.

Vậy $I = e - \frac{2}{3} - \frac{2\sqrt{2}}{3}$.

6. $I = \int\limits_1^2 \frac{\ln(x^2+1)}{x^3}dx$

Đặt $\begin{cases} u = \ln(x^2+1) \\ dv = \dfrac{dx}{x^3} \end{cases} \Rightarrow \begin{cases} du = \dfrac{2x}{x^2+1} \\ v = -\dfrac{1}{2x^2} \end{cases}$. *Do đó I =* $-\left.\frac{\ln(x^2+1)}{2x^2}\right|_1^2 + \int\limits_1^2 \frac{dx}{x(x^2+1)}$

$= \frac{\ln 2}{2} - \frac{\ln 5}{8} + \int\limits_1^2 \left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx = \frac{\ln 2}{2} - \frac{\ln 5}{8} + \int\limits_1^2 \frac{dx}{x} - \frac{1}{2}\int\limits_1^2 \frac{d(x^2+1)}{x^2+1}$

$= \frac{\ln 2}{2} - \frac{\ln 5}{8} + \left.\left(\ln|x| - \frac{1}{2}\ln|x^2+1|\right)\right|_1^2 = 2\ln 2 - \frac{5}{8}\ln 5$

7. I = $\int\limits_1^2 \frac{\ln(x+1)}{x^2}$dx

Đặt $\begin{cases} u = \ln(x+1) \\ dv = \dfrac{dx}{x^2} \end{cases} \Leftrightarrow \begin{cases} du = \dfrac{dx}{x+1} \\ v = -\dfrac{1}{x} \end{cases} \Rightarrow I = -\left.\frac{1}{x}\ln(x+1)\right|_1^2 + \int\limits_1^2 \frac{dx}{(x+1)x} = 3\ln 2 - \frac{3}{2}\ln 3$

8. $I = \int\limits_0^{\frac{1}{2}} x\ln\left(\frac{1+x}{1-x}\right)dx$

Đặt $\begin{cases} u = \ln\dfrac{1+x}{1-x} \\ dv = xdx \end{cases} \Rightarrow \begin{cases} du = \dfrac{2}{(1-x)^2}dx \\ v = \dfrac{x^2}{2} \end{cases} \Rightarrow I = \frac{1}{2}\left[\left.x^2\ln\left(\frac{1+x}{1-x}\right)\right|_0^{\frac{1}{2}} - \int\limits_0^{\frac{1}{2}} x^2\left(\frac{2}{1-x^2}\right)dx\right]$

$$= \frac{\ln 3}{8} + \int_0^{\frac{1}{2}} \frac{x^2}{x^2-1}dx = \frac{\ln 3}{8} + \int_0^{\frac{1}{2}} \left[1 + \frac{1}{(x-1)(x+1)}\right]dx = \frac{\ln 3}{8} + \frac{1}{2} + \frac{1}{2}\ln\frac{2}{3}$$

9. $I = \int_1^2 x^2.\ln\left(x + \frac{1}{x}\right)dx$

Đặt $\begin{cases} u = \ln\left(x + \frac{1}{x}\right) \\ dv = x^2 dx \end{cases} \Rightarrow I = 3\ln 3 - \frac{10}{3}\ln 2 + \frac{1}{6}$

10. $I = \int_0^1 x^2.\ln(1+x^2)dx$

Đặt $\begin{cases} u = \ln(1+x^2) \\ dv = x^2 dx \end{cases} \Rightarrow I = \frac{1}{3}.\ln 2 + \frac{4}{9} + \frac{\pi}{6}$

11. $I = \int_1^3 \frac{\ln x}{(x+1)^2}dx$

Đặt $\begin{cases} u = \ln x \\ dv = \frac{dx}{(x+1)^2} \end{cases} \Rightarrow \quad I = -\frac{1}{4}\ln 3 + \ln\frac{3}{2}$

12. $I = \int_1^e \frac{\ln^2 x + e^x(e^x + \ln^2 x)}{1+e^x}.dx$

Ta có: $I = \int_1^e \ln^2 x.dx + \int_1^e \frac{e^{2x}}{e^x+1}dx = H + K$

+ $H = \int_1^e \ln^2 x.dx$. *Đặt:* $\begin{cases} u = \ln^2 x \\ dv = dx \end{cases} \Rightarrow H = e - \int_1^e 2\ln x.dx = e - 2$

+ $K = \int_1^e \frac{e^{2x}}{e^x+1}dx$. *Đặt* $t = e^x + 1 \Rightarrow \Rightarrow I_2 = \int_{e+1}^{e^e+1} \frac{t-1}{t}dt = e^e - e + \ln\frac{e+1}{e^e+1}$

Vậy: $I = e^e - 2 + \ln\frac{e+1}{e^e+1}$

13. $I = \int_{\frac{1}{2}}^2 (x+1-\frac{1}{x})e^{x+\frac{1}{x}}dx$

Ta có: $I = \int\limits_{\frac{1}{2}}^{2} e^{x+\frac{1}{x}}dx + \int\limits_{\frac{1}{2}}^{3}\left(x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx = H + K$

+ Tính H theo phương pháp từng phần $I_1 =$ $H = xe^{x+\frac{1}{x}}\Big|_{\frac{1}{2}}^{2} - \int\limits_{\frac{1}{2}}^{2}\left(x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx = \frac{3}{2}e^{\frac{5}{2}} - K$

$\Rightarrow I = \frac{3}{2}e^{\frac{5}{2}}.$

14. $I = \int\limits_{0}^{4} \ln(\sqrt{x^2+9} - x)dx$

Đặt $\begin{cases} u = \ln\left(\sqrt{x^2+9} - x\right) \\ dv = dx \end{cases} \Rightarrow I = x\ln\left(\sqrt{x^2+9} - x\right)\Big|_0^4 + \int\limits_0^4 \frac{x}{\sqrt{x^2+9}}dx = 2$

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PHẦN VI TỔNG HỢP

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HT 1.Tính các tích phân sau:

1. $I=\int\limits_0^1\left(x^2e^{x^3}+\frac{\sqrt[4]{x}}{1+\sqrt{x}}\right)dx$	**6.** $I=\int\frac{x\ln(x^2+1)+x^3}{x^2+1}dx$
2. $I=\int\limits_1^2 x\left(e^x-\frac{\sqrt{4-x^2}}{x^3}\right)dx$	**7.** $I=\int\limits_0^4\frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx$
3. $I=\int\limits_0^1\frac{x}{\sqrt{4-x^2}}\left(e^{2x}.\sqrt{4-x^2}-x^2\right)dx.$	**8.** $I=\int\limits_1^e\frac{(x^3+1)\ln x+2x^2+1}{2+x\ln x}dx$
4. $I=\int\limits_0^1\frac{x^2+1}{(x+1)^2}e^xdx$	**9.** $I=\int\limits_1^{e^3}\frac{\ln^3x}{x\sqrt{1+\ln x}}dx$
5. $I=\int\limits_0^{\sqrt3}\frac{x^3.e^{\sqrt{x^2+1}}dx}{\sqrt{1+x^2}}$	**10.** $I=\int\limits_0^{\frac{\pi}{4}}\frac{x\sin x}{\cos^2x}dx$

1. $I=\int\limits_0^1\left(x^2e^{x^3}+\frac{\sqrt[4]{x}}{1+\sqrt{x}}\right)dx$

$I=\int\limits_0^1 x^2e^{x^3}dx+\int\limits_0^1\frac{\sqrt[4]{x}}{1+\sqrt{x}}dx.$

+ *Tính* $I_1=\int\limits_0^1 x^2e^{x^3}dx$. *Đặt* $t=x^3\Rightarrow I_1=\frac{1}{3}\int\limits_0^1 e^tdt=\frac{1}{3}e^t\Big|_0^1=\frac{1}{3}e-\frac{1}{3}.$

+ *Tính* $I_2=\int\limits_0^1\frac{\sqrt[4]{x}}{1+\sqrt{x}}dx$. *Đặt* $t=\sqrt[4]{x}\Rightarrow I_2=4\int\limits_0^1\frac{t^4}{1+t^2}dt=4\left(-\frac{2}{3}+\frac{\pi}{4}\right)$

Vậy: $I=\frac{1}{3}e+\pi-3$

2. $I=\int\limits_1^2 x\left(e^x-\frac{\sqrt{4-x^2}}{x^3}\right)dx$

$I=\int\limits_1^2 xe^xdx+\int\limits_1^2\frac{\sqrt{4-x^2}}{x^2}dx.$

+ *Tính* $I_1=\int\limits_1^2 xe^xdx=e^2$ + *Tính* $I_2=\int\limits_1^2\frac{\sqrt{4-x^2}}{x^2}dx$. *Đặt* $x=2\sin t,\ t\in\left[0;\frac{\pi}{2}\right].$

$\Rightarrow I_2=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos^2t}{\sin^2t}dt=(-\cot t-t)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\sqrt3-\frac{\pi}{3}$

Vậy: $I = e^2 + \sqrt{3} - \frac{\pi}{3}$.

3. $I = \int_0^1 \frac{x}{\sqrt{4-x^2}}\left(e^{2x}.\sqrt{4-x^2} - x^2\right)dx.$

$I = \int_0^1 x\,e^{2x}\,dx - \int_0^1 \frac{x^3}{\sqrt{4-x^2}}\,dx = I_1 + I_2$

+ *Tính* $I_1 = \int_0^1 x\,e^{2x}\,dx = \frac{e^2+1}{4}$

+ *Tính* $I_2 = \int_0^1 \frac{x^3}{\sqrt{4-x^2}}\,dx$. *Đặt* $t = \sqrt{4-x^2} \Rightarrow I_2 = -3\sqrt{3} + \frac{16}{3}$

$\Rightarrow I = \frac{e^2}{4} + 3\sqrt{3} - \frac{61}{12}$

4. $I = \int_0^1 \frac{x^2+1}{(x+1)^2}e^x dx$

Đặt $t = x+1 \Rightarrow dx = dt$ $I = \int_1^2 \frac{t^2-2t+2}{t^2}e^{t-1}dt = \int_1^2 \left(1 + \frac{2}{t^2} - \frac{2}{t}\right)e^{t-1}dt = e - 1 + \frac{2}{e}\left(-\frac{e^2}{2} + e\right) = 1$

5. $I = \int_0^{\sqrt{3}} \frac{x^3.e^{\sqrt{x^2+1}}dx}{\sqrt{1+x^2}}$

Đặt $t = \sqrt{1+x^2} \Rightarrow dx = tdt \Rightarrow I = \int_1^2 (t^2-1)e^t dt = \int_1^2 t^2e^t dt - e^t\Big|_1^2 = J - (e^2 - e)$

+ $J = \int_1^2 t^2e^t dt = t^2e^t\Big|_1^2 - \int_1^2 2te^t dt = 4e^2 - e - 2\left(te^t\Big|_1^2 - \int_1^2 e^t dt\right) = 4e^2 - e - 2(te^t - e^t)\Big|_1^2$

Vậy: $I = e^2$

6. $I = \int \frac{x\ln(x^2+1) + x^3}{x^2+1}dx$

Ta có: $f(x) = \frac{x\ln(x^2+1)}{x^2+1} + \frac{x(x^2+1) - x}{x^2+1} = \frac{x\ln(x^2+1)}{x^2+1} + x - \frac{x}{x^2+1}$

$\Rightarrow F(x) = \int f(x)dx = \frac{1}{2}\int \ln(x^2+1)d(x^2+1) + \int xdx - \frac{1}{2}\int d\ln(x^2+1)$

$= \frac{1}{4}\ln^2(x^2+1) + \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + C$.

7. $I = \int_0^4 \frac{\ln\left(x + \sqrt{x^2+9}\right) - 3x^3}{\sqrt{x^2+9}}dx$

$$I=\int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx=\int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)}{\sqrt{x^2+9}}dx-3\int_0^4 \frac{x^3}{\sqrt{x^2+9}}dx=I_1-3I_2$$

+ Tính $I_1=\int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)}{\sqrt{x^2+9}}dx$. *Đặt* $\ln\left(x+\sqrt{x^2+9}\right)=u \Rightarrow du=\frac{1}{\sqrt{x^2+9}}dx$

$$\Rightarrow I_1=\int_{\ln 3}^{\ln 9} udu=\frac{u^2}{2}\Big|_{\ln 3}^{\ln 9}=\frac{\ln^2 9-\ln^2 3}{2}$$

+ Tính $I_2=\int_0^4 \frac{x^3}{\sqrt{x^2+9}}dx$. *Đặt* $\sqrt{x^2+9}=v \Rightarrow dv=\frac{x}{\sqrt{x^2+9}}dx,\ \ x^2=v^2-9$

$$\Rightarrow I_2=\int_3^5 (u^2-9)du=(\frac{u^3}{3}-9u)\Big|_3^5=\frac{44}{3}$$

Vậy $I=\int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx=I_1-3I_2=\frac{\ln^2 9-\ln^2 3}{2}-44.$

8. $I=\int_1^e \frac{(x^3+1)\ln x+2x^2+1}{2+x\ln x}dx$

$I=\int_1^e x^2dx+\int_1^e \frac{1+\ln x}{2+x\ln x}dx.$ *+* $\int_1^e x^2dx=\frac{x^3}{3}\Big|_1^e=\frac{e^3-1}{3}$

+ $\int_1^e \frac{1+\ln x}{2+x\ln x}dx=\int_1^e \frac{d(2+x\ln x)}{2+x\ln x}=\ln\left|2+x\ln x\right|\Big|_1^e=\ln\frac{e+2}{2}.$ *Vậy:* $I=\frac{e^3-1}{3}+\ln\frac{e+2}{2}.$

9. $I=\int_1^{e^3} \frac{\ln^3 x}{x\sqrt{1+\ln x}}dx$

Đặt $t=\sqrt{1+\ln x}\Rightarrow 1+\ln x=t^2 \Rightarrow \frac{dx}{x}=2tdt$ *và* $\ln^3 x=(t^2-1)^3$

$$\Rightarrow I=\int_1^2 \frac{(t^2-1)^3}{t}dt=\int_1^2 \frac{t^6-3t^4+3t^2-1}{t}dt=\int_1^2 (t^5-3t^3+3t-\frac{1}{t})dt=\frac{15}{4}-\ln 2$$

10. $I=\int_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^2 x}dx$

Đặt $\begin{cases} u=x \\ dv=\frac{\sin x}{\cos^2 x}dx \end{cases} \Rightarrow \begin{cases} du=dx \\ v=\frac{1}{\cos x} \end{cases} \Rightarrow I=\frac{x}{\cos x}\Big|_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} \frac{dx}{\cos x}=\frac{\pi\sqrt{2}}{4}-\int_0^{\frac{\pi}{4}} \frac{dx}{\cos x}$

+ $I_1 = \int\limits_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \int\limits_0^{\frac{\pi}{4}} \frac{\cos x dx}{1-\sin^2 x}$. *Đặt* $t = \sin x \Rightarrow I_1 = \int\limits_0^{\frac{\sqrt{2}}{2}} \frac{dt}{1-t^2} = \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

Vậy: $= \frac{\pi\sqrt{2}}{4} - \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

HT 2.Tính các tích phân sau:

1. $I = \int\limits_1^4 \frac{\ln(5-x) + x^3.\sqrt{5-x}}{x^2}dx$	**7.** $I = \int\limits_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^3 x}dx$
2. $I = \int\limits_0^2 \left[\sqrt{x(2-x)} + \ln(4+x^2)\right]dx$	**8.** $I = \int\limits_0^{\frac{\pi}{2}} \frac{(x+\sin^2 x)}{1+\sin 2x}dx$
3. $I = \int\limits_3^8 \frac{\ln x}{\sqrt{x+1}}dx$	**9.** $I = \int\limits_0^{\pi} \frac{x(\cos^3 x + \cos x + \sin x)}{1+\cos^2 x}dx$
4. $I = \int\limits_1^2 \frac{1+x^2}{x^3}\ln x dx$	**10.** $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x+\sin x)\sin x}{(1+\sin x)\sin^2 x}dx$
5. $I = \int\limits_1^e \frac{x^2 + x\ln x + 1}{x}e^x dx$	
6. $I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\cos x}{\sin^3 x}dx$	

Bài giải

1. $I = \int\limits_1^4 \frac{\ln(5-x) + x^3.\sqrt{5-x}}{x^2}dx$

Ta có: $I = \int\limits_1^4 \frac{\ln(5-x)}{x^2}dx + \int\limits_1^4 x\sqrt{5-x}.dx = K + H$.

+ $K = \int\limits_1^4 \frac{\ln(5-x)}{x^2}dx$. *Đặt* $\begin{cases} u = \ln(5-x) \\ dv = \frac{dx}{x^2} \end{cases} \Rightarrow K = \frac{3}{5}\ln 4$

+ $H = \int\limits_1^4 x\sqrt{5-x}.dx$. *Đặt* $t = \sqrt{5-x} \Rightarrow H = \frac{164}{15}$

Vậy: $I = \frac{3}{5}\ln 4 + \frac{164}{15}$

2. $I = \int\limits_0^2 \left[\sqrt{x(2-x)} + \ln(4+x^2)\right]dx$

Ta có: $I = \int\limits_0^2 \sqrt{x(2-x)}dx + \int\limits_0^2 \ln(4+x^2)dx = I_1 + I_2$

+ $I_1 = \int\limits_0^2 \sqrt{x(2-x)}dx = \int\limits_0^2 \sqrt{1-(x-1)^2}dx = \frac{\pi}{2}$ *(sử dụng đổi biến:* $x = 1 + \sin t$ *)*

+ $I_2 = \int\limits_0^2 \ln(4+x^2)dx = x\ln(4+x^2)\Big|_0^2 - 2\int\limits_0^2 \frac{x^2}{4+x^2}dx$ *(sử dụng tích phân từng phần)*

$= 6\ln 2 + \pi - 4$ *(đổi biến* $x = 2\tan t$ *)*

Vậy: $I = I_1 + I_2 = \frac{3\pi}{2} - 4 + 6\ln 2$

3. $I = \int\limits_3^8 \frac{\ln x}{\sqrt{x+1}}dx$

Đặt $\begin{cases} u = \ln x \\ dv = \frac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \frac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = 2\sqrt{x+1}\ln x\Big|_3^8 - 2\int\limits_3^8 \frac{\sqrt{x+1}}{x}dx$

+ *Tính* $J = \int\limits_3^8 \frac{\sqrt{x+1}}{x}dx$. *Đặt* $t = \sqrt{x+1} \Rightarrow J = \int\limits_2^3 \frac{2t^2dt}{t^2-1} = 2\int\limits_2^3 \left(1 + \frac{1}{t^2-1}\right)dt = 2 + \ln 3 - \ln 2$

$\Rightarrow I = 6\ln 8 - 4\ln 3 - 2(2 + \ln 3 - \ln 2) = 20\ln 2 - 6\ln 3 - 4$

4. $I = \int\limits_1^2 \frac{1+x^2}{x^3}\ln x dx$

Ta có: $I = \int\limits_1^2 \left(\frac{1}{x^3} + \frac{1}{x}\right)\ln x dx$. *Đặt* $\begin{cases} u = \ln x \\ dv = (\frac{1}{x^3} + \frac{1}{x})dx \end{cases}$

$\Rightarrow I = \left(\frac{-1}{4x^4} + \ln x\right)\ln x\Big|_1^2 - \int\limits_1^2 \left(\frac{-1}{4x^5} + \frac{1}{x}\ln x\right)dx = -\frac{1}{64}\ln 2 + \frac{63}{4} + \frac{1}{2}\ln^2 2$

5. $I = \int\limits_1^e \frac{x^2 + x\ln x + 1}{x}e^x dx$

Ta có: $I = \int\limits_1^e xe^x dx + \int\limits_1^e e^x \ln x dx + \int\limits_1^e \frac{e^x}{x}dx = H + K + J$

+ $H = \int\limits_1^e xe^x dx = xe^x\Big|_1^e - \int\limits_1^e e^x dx = e^e(e-1)$

+ $K = \int\limits_1^e e^x \ln x dx = e^x \ln x\Big|_1^e - \int\limits_1^e \frac{e^x}{x}dx = e^e - \int\limits_1^e \frac{e^x}{x}dx = e^e - J$

Vậy: $I = H + K + J = e^{e+1} - e^e + e^e - J + J = e^{e+1}$.

6. $I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\cos x}{\sin^3 x}dx$

Ta có $\left(\frac{1}{\sin^2 x}\right)' = -\frac{2\cos x}{\sin^3 x}$. *Đặt* $\begin{cases} u = x \\ dv = \frac{\cos x}{\sin^3 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\frac{1}{2\sin^2 x} \end{cases}$

$\Rightarrow I = -\frac{1}{2}x.\frac{1}{\sin^2 x}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2}\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\sin^2 x} = -\frac{1}{2}(\frac{\pi}{2} - \frac{\pi}{2}) - \frac{1}{2}\cot x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2}$.

7. $I = \int\limits_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^3 x}dx$

Đặt: $\begin{cases} u = x \\ dv = \frac{\sin x}{\cos^3 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{2.\cos^2 x} \end{cases} \Rightarrow I = \frac{x}{2\cos^2 x}\Big|_0^{\frac{\pi}{4}} - \frac{1}{2}\int\limits_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \frac{\pi}{4} - \frac{1}{2}\tan x\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \frac{1}{2}$

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8. $I = \int\limits_0^{\frac{\pi}{2}} \frac{(x + \sin^2 x)}{1 + \sin 2x}dx$

Ta có: $I = \int\limits_0^{\frac{\pi}{2}} \frac{x}{1 + \sin 2x}dx + \int\limits_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x}dx = H + K$

+ $H = \int\limits_0^{\frac{\pi}{2}} \frac{x}{1 + \sin 2x}dx = \int\limits_0^{\frac{\pi}{2}} \frac{x}{2\cos^2\left(x - \frac{\pi}{4}\right)}dx$. *Đặt:* $\begin{cases} u = x \\ dv = \frac{dx}{2\cos^2\left(x - \frac{\pi}{4}\right)} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{2}\tan\left(x - \frac{\pi}{4}\right) \end{cases}$

$\Rightarrow H = \frac{x}{2}\tan\left(x - \frac{\pi}{4}\right)\Big|_0^{\frac{\pi}{2}} + \left(\frac{1}{2}\ln\left|\cos\left(x - \frac{\pi}{4}\right)\right|\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

+ $K = \int\limits_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x}dx$. *Đặt* $t = \frac{\pi}{2} - x \Rightarrow K = \int\limits_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + \sin 2x}dx$

$$\Rightarrow 2K=\int_0^{\frac{\pi}{2}}\frac{dx}{2\cos^2\left(x-\frac{\pi}{4}\right)}=\frac{1}{2}\tan\left(x-\frac{\pi}{4}\right)\Big|_0^{\frac{\pi}{2}}=1\ \Rightarrow K=\frac{1}{2}$$

Vậy, $I=H+K=\frac{\pi}{4}+\frac{1}{2}$.

9. $I=\int_0^{\pi}\frac{x(\cos^3 x+\cos x+\sin x)}{1+\cos^2 x}dx$

Ta có: $I=\int_0^{\pi}x\left(\frac{\cos x(1+\cos^2 x)+\sin x}{1+\cos^2 x}\right)dx=\int_0^{\pi}x.\cos x.dx+\int_0^{\pi}\frac{x.\sin x}{1+\cos^2 x}dx=J+K$

+ Tính $J=\int_0^{\pi}x.\cos x.dx$ *. Đặt* $\begin{cases}u=x\\ dv=\cos xdx\end{cases}\Rightarrow J=(x.\sin x)\Big|_0^{\pi}-\int_0^{\pi}\sin x.dx=0+\cos x\Big|_0^{\pi}=-2$

+ Tính $K=\int_0^{\pi}\frac{x.\sin x}{1+\cos^2 x}dx$ *. Đặt* $x=\pi-t\Rightarrow dx=-dt$

$$\Rightarrow K=\int_0^{\pi}\frac{(\pi-t).\sin(\pi-t)}{1+\cos^2(\pi-t)}dt=\int_0^{\pi}\frac{(\pi-t).\sin t}{1+\cos^2 t}dt=\int_0^{\pi}\frac{(\pi-x).\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow 2K=\int_0^{\pi}\frac{(x+\pi-x).\sin x}{1+\cos^2 x}dx=\pi\int_0^{\pi}\frac{\sin x.dx}{1+\cos^2 x}\Rightarrow K=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x.dx}{1+\cos^2 x}$$

Đặt $t=\cos x\Rightarrow dt=-\sin x.dx\ \Rightarrow K=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}$, *đặt* $t=\tan u\Rightarrow dt=(1+\tan^2 u)du$

$$\Rightarrow K=\frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan^2 u)du}{1+\tan^2 u}=\frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}du=\frac{\pi}{2}.u\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}=\frac{\pi^2}{4}$$

Vậy $I=\frac{\pi^2}{4}-2$

10. $I=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{x+(x+\sin x)\sin x}{(1+\sin x)\sin^2 x}dx$

Ta có: $I=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{x(1+\sin x)+\sin^2 x}{(1+\sin x)\sin^2 x}dx=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{x}{\sin^2 x}dx+\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{dx}{1+\sin x}=H+K$

+ $H=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{x}{\sin^2 x}dx$. Đặt $\begin{cases}u=x\\ dv=\dfrac{dx}{\sin^2 x}\end{cases}\Rightarrow\begin{cases}du=dx\\ v=-\cot x\end{cases}\Rightarrow H=\frac{\pi}{\sqrt{3}}$

+ $K=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{dx}{1+\sin x}=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)}=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{dx}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}=\sqrt{3}-2$

Vậy $I=\frac{\pi}{\sqrt{3}}+\sqrt{3}-2$

HT 3.Tính các tích phân sau:

1. $I=\int_0^{\frac{\pi}{3}}\frac{x+\sin^2 x}{1+\cos 2x}dx$	**6.** $I=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{\sin^4 xdx}{2^{-x}+1}$
2. $I=\int_0^3\sqrt{x+1}\sin\sqrt{x+1}.dx$	**7.** $I=\int_1^{e^{\pi}}\cos(\ln x)dx$
3. $I=\int_0^{\frac{\pi}{2}}\frac{1+\sin x}{1+\cos x}.e^x dx$	**8.** $I=\int_0^{\frac{\pi}{2}}e^{\sin^2 x}.\sin x.\cos^3 xdx$
4. $I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{e^x(1+\sin 2x)}dx$	**9.** $I=\int_0^{\frac{\pi}{4}}\ln(1+\tan x)dx$
5. $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin^6 x+\cos^6 x}{6^x+1}dx$	**10.** $I=\int_0^{\frac{\pi}{2}}\sin x\ln(1+\sin x)dx$

Bài giải

1. $I=\int_0^{\frac{\pi}{3}}\frac{x+\sin^2 x}{1+\cos 2x}dx$

Ta có: $I=\int_0^{\frac{\pi}{3}}\frac{x+\sin^2 x}{1+\cos 2x}dx=\int_0^{\frac{\pi}{3}}\frac{x}{2\cos^2 x}dx+\int_0^{\frac{\pi}{3}}\frac{\sin^2 x}{2\cos^2 x}dx=H+K$

+ $H=\int_0^{\frac{\pi}{3}}\frac{x}{2\cos^2 x}dx=\frac{1}{2}\int_0^{\frac{\pi}{3}}\frac{x}{\cos^2 x}dx$. Đặt $\begin{cases}u=x\\ dv=\dfrac{dx}{\cos^2 x}\end{cases}\Rightarrow\begin{cases}du=dx\\ v=\tan x\end{cases}$

$$\Rightarrow H=\frac{1}{2}\left[x\tan x\Big|_0^{\frac{\pi}{3}}-\int_0^{\frac{\pi}{3}}\tan xdx\right]=\frac{\pi}{2\sqrt{3}}+\frac{1}{2}\ln\left|\cos x\right|\Big|_0^{\frac{\pi}{3}}=\frac{\pi}{2\sqrt{3}}-\frac{1}{2}\ln 2$$

+ $K=\int_0^{\frac{\pi}{3}}\frac{\sin^2 x}{2\cos^2 x}dx=\frac{1}{2}\int_0^{\frac{\pi}{3}}\tan^2 xdx=\frac{1}{2}\left[\tan x-x\right]\Big|_0^{\frac{\pi}{3}}=\frac{1}{2}\left(\sqrt{3}-\frac{\pi}{3}\right)$

Vậy: $I = H + K = \dfrac{\pi}{2\sqrt{3}} - \dfrac{1}{2}\ln 2 + \dfrac{1}{2}\left(\sqrt{3} - \dfrac{\pi}{3}\right) = \dfrac{\pi\left(\sqrt{3}-1\right)}{6} + \dfrac{1}{2}(\sqrt{3} - \ln 2)$

2. $I = \displaystyle\int_0^3 \sqrt{x+1}\sin\sqrt{x+1}.dx$

Đặt $t = \sqrt{x+1} \Rightarrow I = \displaystyle\int_1^2 t.\sin t.2tdt = \int_1^2 2t^2\sin tdt = \int_1^2 2x^2\sin xdx$

Đặt $\begin{cases} u = 2x^2 \\ dv = \sin xd\text{x} \end{cases} \Rightarrow \begin{cases} du = 4xd\text{x} \\ v = -\cos x \end{cases} \Rightarrow I = -2x^2\cos x\Big|_1^2 + \displaystyle\int_1^2 4x\cos xdx$

Đặt $\begin{cases} u = 4x \\ dv = \cos xd\text{x} \end{cases} \Rightarrow \begin{cases} du = 4d\text{x} \\ v = \sin x \end{cases}$. *Từ đó suy ra kết quả.*

3. $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{1+\sin x}{1+\cos x}.e^x dx$

$I = \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{2}} \frac{e^x dx}{\cos^2\frac{x}{2}} + \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}e^x dx$

+ Tính $I_1 = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}e^x dx = \int_0^{\frac{\pi}{2}} \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}e^x dx = \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}e^x dx$

+ Tính $I_2 = \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{2}} \frac{e^x dx}{\cos^2\frac{x}{2}}$. *Đặt* $\begin{cases} u = e^x \\ dv = \dfrac{dx}{2\cos^2\frac{x}{2}} \end{cases} \Rightarrow \begin{cases} du = e^x dx \\ v = \tan\dfrac{x}{2} \end{cases} \Rightarrow I_2 = e^{\frac{\pi}{2}} - \displaystyle\int_0^{\frac{\pi}{2}} \tan\frac{x}{2}e^x dx$

Do đó: $I = I_1 + I_2 = e^{\frac{\pi}{2}}$.

4. $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x(1+\sin 2x)}dx$

$I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x(\sin x+\cos x)^2}dx$. *Đặt* $\begin{cases} u = \dfrac{\cos x}{e^x} \\ dv = \dfrac{dx}{(\sin x+\cos x)^2} \end{cases} \Rightarrow \begin{cases} du = \dfrac{-(\sin x+\cos x)dx}{e^x} \\ v = \dfrac{\sin x}{\sin x+\cos x} \end{cases}$

$\Rightarrow I = \dfrac{\cos x}{e^x}.\dfrac{\sin x}{\sin x+\cos x}\Big|_0^{\frac{\pi}{2}} + \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin xdx}{e^x} = \int_0^{\frac{\pi}{2}} \frac{\sin xdx}{e^x}$

Đặt $\begin{cases} u_1 = \sin x \\ dv_1 = \dfrac{dx}{e^x} \end{cases} \Rightarrow \begin{cases} du_1 = \cos x dx \\ v_1 = \dfrac{-1}{e^x} \end{cases} \Rightarrow I = \sin x.\dfrac{-1}{e^x}\Bigg|_0^{\frac{\pi}{2}} + \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\cos x dx}{e^x} = \dfrac{-1}{e^{\frac{\pi}{2}}} + \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\cos x dx}{e^x}$

Đặt $\begin{cases} u_2 = \cos x \\ dv_1 = \dfrac{dx}{e^x} \end{cases} \Rightarrow \begin{cases} du_2 = -\sin x dx \\ v_1 = \dfrac{-1}{e^x} \end{cases}$

$$\Rightarrow I = \frac{-1}{e^{\frac{\pi}{2}}} + \cos x.\frac{-1}{e^x}\Bigg|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x} = \frac{-1}{e^{\frac{\pi}{2}}} + 1 - I \Rightarrow 2I = -e^{\frac{-\pi}{2}} + 1 \Rightarrow I = \frac{-e^{\frac{-\pi}{2}}}{2} + \frac{1}{2}$$

5. $I = \displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \dfrac{\sin^6 x + \cos^6 x}{6^x + 1} dx$

Đặt $t = -x \Rightarrow dt = -dx \Rightarrow I = \displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^t \dfrac{\sin^6 t + \cos^6 t}{6^t + 1} dt = \displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^x \dfrac{\sin^6 x + \cos^6 x}{6^x + 1} dx$

$$\Rightarrow 2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (6^x + 1)\frac{\sin^6 x + \cos^6 x}{6^x + 1} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin^6 x + \cos^6 x) dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{5}{8} + \frac{3}{8}\cos 4x\right) dx = \frac{5\pi}{16}$$

$\Rightarrow I = \dfrac{5\pi}{32}.$

6. $I = \displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \dfrac{\sin^4 x dx}{2^{-x} + 1}$

Ta có: $I = \displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \dfrac{2^x \sin^4 x dx}{2^x + 1} = \displaystyle\int_{-\frac{\pi}{6}}^{0} \dfrac{2^x \sin^4 x dx}{2^x + 1} + \displaystyle\int_0^{\frac{\pi}{6}} \dfrac{2^x \sin^4 x dx}{2^x + 1} = I_1 + I_2$

+ Tính $I_1 = \displaystyle\int_{-\frac{\pi}{6}}^{0} \dfrac{2^x \sin^4 x dx}{2^x + 1}$. *Đặt* $x = -t \Rightarrow I_1 = -\displaystyle\int_{\frac{\pi}{6}}^{0} \dfrac{2^{-t} \sin^4(-t)}{2^{-t} + 1} dt = \displaystyle\int_{\frac{\pi}{6}}^{0} \dfrac{\sin^4 t}{2^t + 1} dt = \displaystyle\int_{\frac{\pi}{6}}^{0} \dfrac{\sin^4 x}{2^x + 1} dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{6}} \frac{\sin^4 x dx}{2^x + 1} + \int_0^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = \int_0^{\frac{\pi}{6}} \sin^4 x dx = \frac{1}{4}\int_0^{\frac{\pi}{6}} (1 - \cos 2x)^2 dx$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{6}}(3 - 4\cos 2x + \cos 4x)dx = \frac{4\pi - 7\sqrt{3}}{64}$$

7. $I = \int_1^{e^\pi} \cos(\ln x)dx$

Đặt $t = \ln x \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$\Rightarrow I = \int_0^{\pi} e^t \cos t dt = -\frac{1}{2}(e^\pi + 1)$ *(dùng pp tích phân từng phần).*

8. $I = \int_0^{\frac{\pi}{2}} e^{\sin^2 x}.\sin x.\cos^3 x dx$

Đặt $t = \sin^2 x \Rightarrow I = \frac{1}{2}\int_0^1 e^t(1-t)dt = \frac{1}{2}e$ *(dùng tích phân từng phần)*

9. $I = \int_0^{\frac{\pi}{4}} \ln(1 + \tan x)dx$

Đặt $t = \frac{\pi}{4} - x \Rightarrow I = \int_0^{\frac{\pi}{4}} \ln\left(1 + \tan\left(\frac{\pi}{4} - t\right)\right)dt = \int_0^{\frac{\pi}{4}} \ln\left(1 + \frac{1-\tan t}{1+\tan t}\right)dt = \int_0^{\frac{\pi}{4}} \ln\frac{2}{1+\tan t}dt$

$= \int_0^{\frac{\pi}{4}} \ln 2 dt - \int_0^{\frac{\pi}{4}} \ln(1+\tan t)dt = t.\ln 2\Big|_0^{\frac{\pi}{4}} - I$

$\Rightarrow 2I = \frac{\pi}{4}\ln 2 \Rightarrow I = \frac{\pi}{8}\ln 2.$

10. $I = \int_0^{\frac{\pi}{2}} \sin x \ln(1 + \sin x)dx$

Đặt $\begin{cases} u = \ln(1+\sin x) \\ dv = \sin x dx \end{cases} \Rightarrow \begin{cases} du = \dfrac{1+\cos x}{1+\sin x}dx \\ v = -\cos x \end{cases}$

$\Rightarrow I = -\cos x.\ln(1+\sin x)\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x.\frac{\cos x}{1+\sin x}dx = 0 + \int_0^{\frac{\pi}{2}} \frac{1-\sin^2 x}{1+\sin x}dx = \int_0^{\frac{\pi}{2}} (1 - \sin x)dx = \frac{\pi}{2} - 1$

11. $I = \int_{0}^{\frac{\pi}{4}} \frac{\tan x.\ln(\cos x)}{\cos x}dx$

Đặt $t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow I = -\int_{1}^{\frac{1}{\sqrt{2}}} \frac{\ln t}{t^2}dt = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{\ln t}{t^2}dt$.

Đặt $\begin{cases} u = \ln t \\ dv = \dfrac{1}{t^2}dt \end{cases} \Rightarrow \begin{cases} du = \dfrac{1}{t}dt \\ v = -\dfrac{1}{t} \end{cases} \Rightarrow I = \sqrt{2} - 1 - \frac{\sqrt{2}}{2}\ln 2$

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PHẦN VII TUYỂN TẬP MỘT SỐ ĐỀ THI THỬ

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HT 1.Tính các tích phân sau:

1. $I=\int\limits_1^e\left(\frac{\ln x}{x\sqrt{1+\ln x}}+\ln^2 x\right)dx$	**10.** $\int\limits_1^e \frac{x\ln x+\ln(x.e^2)}{x\ln x+1}\,dx$
2. $\int\limits_1^e \frac{\ln(1+\ln^2 x)}{x}dx$	**11.** $I=\int\limits_1^e \frac{\left(x^3+1\right)\ln x+2x^2+1}{2+x\ln x}dx$
3. $\int\limits_1^e \frac{\ln x-2}{x\ln x+x}dx$	**12.** $A=\int_0^{\frac{\pi}{2}}\sin x\cos x\ln\left(1+\sin^2 x\right)dx$
4. $I=\int\limits_1^e\left(\frac{\ln x}{x\sqrt{1+\ln x}}+3x^2\ln x\right)dx$	**13.** $\int\limits_1^e \frac{\ln x-2}{x\ln x+x}dx$
5. $\int\limits_1^e \frac{(x-2)\ln x+x}{x(1+\ln x)}dx$	**14.** $I=\int\limits_{\sqrt{2}}^e x^3\ln\frac{x^2-1}{x^2+1}dx$
6. $\int\limits_{\frac{3\pi}{4}}^{\pi}\left[\frac{e^{\frac{1}{x}}}{x^2}+x\left(\frac{x}{\cos^2 x}+2\tan x\right)\right]dx$	**15.** $\int\limits_1^e \frac{x^2\ln x+x\ln^2 x+x+1}{x^2+x\ln x}dx$
7. $I=\int\limits_{e^2}^{e^3}\frac{2x\ln x\left(\ln x-1\right)+3}{x\left(1-\ln x\right)}dx$	**16.** $I=\int\limits_0^1 x\ln\left(x^2+x+1\right)dx$
8. $I=\int\limits_1^2 \frac{x^3.3^{x^2+1}+\ln(x+1)}{x^2}dx$	**17.** $\int\limits_{\frac{1}{2}}^2 (x+1-\frac{1}{x})e^{x+\frac{1}{x}}dx$
9. $\int\limits_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx$	

Bài giải

1. $I=\int\limits_1^e\left(\frac{\ln x}{x\sqrt{1+\ln x}}+\ln^2 x\right)dx$

$I_1=\int\limits_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx$, Đặt t = $\sqrt{1+\ln x}$,... Tính được $I_1=\frac{4}{3}-\frac{2\sqrt{2}}{3}$

$I_2=\int\limits_1^e\left(\ln^2 x\right)dx$, lấy tích phân từng phần 2 lần được $I_2 = e - 2$ Vậy: $I = I_1 + I_2 = e-\frac{2}{3}-\frac{2\sqrt{2}}{3}$

2. $\int\limits_1^e \frac{\ln(1+\ln^2 x)}{x}dx$

Đặt lnx = t , ta có I = $\int\limits_0^1 \ln(1+t^2)dt$. Đặt u = ln(1+t²) , dv = dt ta có : du = $\frac{2t}{1+t^2}dt$, $v = t$.

Từ đó có : I = t ln(1+ t²) $\Big|_0^1 - 2\int\limits_0^1 \frac{t^2}{1+t^2}dt = \ln 2 - 2\left(\int\limits_0^1 dt - \int\limits_0^1 \frac{dt}{1+t^2}\right)$ (*).

Tiếp tục đặt t = tanu , ta tính được $\int\limits_0^1 \frac{dt}{1+t^2} = \frac{\pi}{4}$ ***.Thay vào (*) ta có : I = ln2 – 2 +*** $\frac{\pi}{2}$ ***.***

3. $\int\limits_1^e \frac{\ln x - 2}{x\ln x + x}dx$ = $\int\limits_1^e \frac{\ln x - 2}{(\ln x + 1)x}dx$

Đặt t = lnx + 1 $\Rightarrow$ dt = $\frac{1}{x}dx$;

Đổi cận: x = 1 thì t = 1; x = e thì t = 2

Suy ra: I = $\int\limits_1^2 \frac{t-3}{t}dt = \int\limits_1^2 \left(1 - \frac{3}{t}\right)dt$ = $\left(t - \ln|t|\right)\Big|_1^2$ = 1 – ln2

4. $I = \int\limits_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + 3x^2\ln x\right)dx$

$I = \int\limits_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx + 3\int\limits_1^e x^2\ln x dx$ $=I_1+3I_2$

+) Tính $I_1 = \int\limits_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx$.

Đặt $t = \sqrt{1+\ln x} \Rightarrow t^2 = 1 + \ln x;\ 2tdt = \frac{1}{x}dx$

Khi $x = 1 \Rightarrow t = 1; x = e \Rightarrow t = \sqrt{2}$

$$\Rightarrow I_1 = \int\limits_1^{\sqrt{2}} \frac{\left(t^2-1\right)}{t}.2tdt = 2\int\limits_1^{\sqrt{2}}\left(t^2-1\right)dt = 2\left(\frac{t^3}{3} - t\right)\Bigg|_1^{\sqrt{2}} = \frac{2\left(2-\sqrt{2}\right)}{3}$$

+)Tính $I_2 = \int\limits_1^e x^2 \ln x dx$. Đặt $\begin{cases} u = \ln x \\ dv = x^2 dx \end{cases} \Rightarrow \begin{cases} du = \dfrac{dx}{x} \\ v = \dfrac{x^3}{3} \end{cases}$

$$\Rightarrow I_2 = \frac{x^3}{3}.\ln x\Big|_1^e - \frac{1}{3}\int\limits_1^e x^2 dx = \frac{e^3}{3} - \frac{1}{3}.\frac{x^3}{3}\Big|_1^e = \frac{e^3}{3} - \frac{e^3}{9} + \frac{1}{9} = \frac{2e^3+1}{9}$$

$$I = I_1 + 3I_2 = \frac{5 - 2\sqrt{2} + 2e^3}{3}$$

5. $\int\limits_1^e \dfrac{(x-2)\ln x + x}{x(1+\ln x)} dx$

I = $\int\limits_1^e \dfrac{x(1+\ln x) - 2\ln x}{x(1+\ln x)} dx = \int\limits_1^e dx$ -2 $\int\limits_1^e \dfrac{\ln x}{x(1+\ln x)} dx$ Ta có : $\int\limits_1^e dx = e - 1$

Tính J = $\int\limits_1^e \dfrac{\ln x}{x(1+\ln x)} dx$

Đặt t = 1 + lnx, Ta có: J = $\int\limits_1^2 \dfrac{t-1}{t} dt$ = $\int\limits_1^2 (1 - \dfrac{1}{t}) dt$ = (t - ln$|t|$) = 1 - ln2

Vậy **I** = e - 1 - 2(1- ln2) = e - 3 + 2ln2

6. $\int\limits_{\frac{3\pi}{4}}^{\pi} \left[\dfrac{e^{\frac{1}{x}}}{x^2} + x\left(\dfrac{x}{\cos^2 x} + 2\tan x\right)\right] dx$

Ta có: $I = \int\limits_{\frac{3\pi}{4}}^{\pi} \left[\dfrac{e^{\frac{1}{x}}}{x^2} + x\left(\dfrac{x}{\cos^2 x} + 2\tan x\right)\right] dx = \int\limits_{\frac{3\pi}{4}}^{\pi} e^{\frac{1}{x}}.\dfrac{1}{x^2} dx + \int\limits_{\frac{3\pi}{4}}^{\pi} \dfrac{x^2}{\cos^2 x} dx + \int\limits_{\frac{3\pi}{4}}^{\pi} 2x\tan x dx$ (1)

+) $\int\limits_{\frac{3\pi}{4}}^{\pi} e^{\frac{1}{x}}.\dfrac{1}{x^2} dx = -\int\limits_{\frac{3\pi}{4}}^{\pi} e^{\frac{1}{x}} d\left(\dfrac{1}{x}\right) = -e^{\frac{1}{x}}\Big|_{\frac{3\pi}{4}}^{\pi} = -e^{\frac{1}{\pi}} + e^{\frac{4}{3\pi}}$

+) $J = \int\limits_{\frac{3\pi}{4}}^{\pi} \dfrac{x^2}{\cos^2 x} dx$: Đặt $\begin{cases} u = x^2 \\ dv = \dfrac{1}{\cos^2 x} dx \end{cases} \Rightarrow \begin{cases} du = 2xdx \\ v = \tan x \end{cases} \Rightarrow J = \left(x^2 \tan x\right)\Big|_{\frac{3\pi}{4}}^{\pi} - \int\limits_{\frac{3\pi}{4}}^{\pi} 2x\tan x dx$

$J=\frac{9\pi^2}{16}-\int\limits_{\frac{3\pi}{4}}^{\pi}2x\tan xdx$ Thay vào (1) ta có $I=-e^{\frac{1}{\pi}}+e^{\frac{4}{3\pi}}+\frac{9\pi^2}{16}$

7. $I=\int\limits_{e^2}^{e^3}\frac{2x\ln x\left(\ln x-1\right)+3}{x\left(1-\ln x\right)}dx=3\int\limits_{e^2}^{e^3}\frac{1}{x\left(1-\ln x\right)}dx-2\int\limits_{e^2}^{e^3}\ln xdx=3\int\limits_{e^2}^{e^3}\frac{1}{\left(1-\ln x\right)}d(\ln x)-2\left(x\ln x\Big|_{e^2}^{e^3}-\int\limits_{e^2}^{e^3}dx\right)$

$=-3\ln\left(\left|1-\ln x\right|\right)\Big|_{e^2}^{e^3}-2\left(x\ln x\Big|_{e^2}^{e^3}-x\Big|_{e^2}^{e^3}\right)=-3\ln 2-4e^3+2e^2.$

$\mathrm{I}=\int\limits_1^e\frac{x^2+x\ln x+1}{x}e^xdx$

I= $\int\limits_1^e\frac{x^2+x\ln x+1}{x}e^xdx=\int\limits_1^e xe^xdx+\int\limits_1^e\ln xe^xdx+\int\limits_1^e\frac{e^x}{x}dx$

Đặt I$_1$= $\int\limits_1^e xe^xdx=xe^x\Big|_1^e-\int\limits_1^e e^xdx=e^e\left(e-1\right)$

Đặt I$_2$= $\int\limits_1^e e^x\ln xdx=e^x\ln x\Big|_1^e-\int\limits_1^e\frac{e^x}{x}dx=e^e-\int\limits_1^e\frac{e^x}{x}dx$

Vậy I=I$_1$+I$_2$+ $\int\limits_1^e\frac{e^x}{x}dx=e^{e+1}-e^e+e^e-\int\limits_1^e\frac{e^x}{x}dx+\int\limits_1^e\frac{e^x}{x}dx=e^{e+1}$

8. $I=\int\limits_1^2\frac{x^3.3^{x^2+1}+\ln(x+1)}{x^2}dx$

Ta có $I=\int\limits_1^2 x.3^{x^2+1}dx+\int\limits_1^2\frac{\ln(x+1)}{x^2}dx=J+K$

Tính: $J=\int\limits_1^2 x.3^{x^2+1}dx=\frac{1}{2}\int\limits_1^2 3^{x^2+1}d(x^2+1)=\frac{3^{x^2+1}}{2\ln 3}\Bigg|_1^2=\frac{117}{\ln 3}.$

Tính: $K=\int\limits_1^2\frac{\ln(x+1)}{x^2}dx$. Đặt $\begin{cases}u=\ln(x+1)\\ v'=\frac{1}{x^2}\end{cases}\Rightarrow\begin{cases}u'=\frac{1}{x+1}\\ v=-\frac{1}{x}\end{cases}.$

Suy ra $K=-\frac{\ln(x+1)}{x}\Bigg|_1^2+\int\limits_1^2\frac{dx}{x(x+1)}=-\frac{\ln 3}{2}+\ln 2+\int\limits_1^2\left(\frac{1}{x}-\frac{1}{x+1}\right)dx=\frac{2\ln 2-\ln 3}{2}+\ln\left|\frac{x}{x+1}\right|\Bigg|_1^2$

$= \dfrac{2\ln 2 - \ln 3}{2} + \ln\dfrac{2}{3} - \ln\dfrac{1}{2} = 3\ln 2 - \dfrac{3}{2}\ln 3.$

Vậy $I = \dfrac{117}{\ln 3} + 3\ln 2 - \dfrac{3}{2}\ln 3$.

9. $\displaystyle\int_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx$

Đặt t = $\sqrt{1+\ln x}$ có 2tdt = $\dfrac{1}{x}dx$ x = 1 thì t = 1; x = e thì t = $\sqrt{2}$

$$\int_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx = \int_1^{\sqrt{2}} \frac{t^2-1}{t}2tdt = 2(\frac{t^3}{3}-t)\Big|_1^{\sqrt{2}} = \frac{2(2-\sqrt{2})}{3}$$

$$\int_1^e \frac{x\ln x + \ln(x.e^2)}{x\ln x + 1}\,dx$$

10.I= $\displaystyle\int_1^e \frac{x\ln x + \ln(x.e^2)}{x\ln x + 1}\,dx$.

$$I = \int_1^e \frac{x\ln x + 1 + \ln x + 1}{x\ln x + 1}\,dx = \int_1^e dx + \int_1^e \frac{\ln x + 1}{x\ln x + 1}\,dx = x\Big|_1^e + \int_1^e \frac{d(x\ln x + 1)}{x\ln x + 1}$$

$$= e - 1 + \ln\left|x\ln x + 1\right|\Big|_1^e = e - 1 + \ln(e+1)$$

11. $I = \displaystyle\int_1^e \frac{\left(x^3+1\right)\ln x + 2x^2 + 1}{2 + x\ln x}dx$

$$I = \int_1^e \frac{\left(x^3+1\right)\ln x + 2x^2 + 1}{2 + x\ln x}dx = \int_1^e x^2dx + \int_1^e \frac{1+\ln x}{2+x\ln x}dx$$

Ta có: $\displaystyle\int_1^e x^2dx = \left(\frac{x^3}{3}\right)\Bigg|_1^e = \frac{e^3-1}{3}$

$$\int_1^e \frac{1+\ln x}{2+x\ln x}dx = \int_1^e \frac{d\left(2+x\ln x\right)}{2+x\ln x} = \left(\ln\left|2+x\ln x\right|\right)\Big|_1^e = \ln\left(e+2\right) - \ln 2 = \ln\frac{e+2}{2}$$

Vậy $I = \dfrac{e^3-1}{3} + \ln\dfrac{e+2}{2}$.

12. $A = \displaystyle\int_0^{\frac{\pi}{2}} \sin x\cos x\ln\left(1+\sin^2 x\right)dx$

$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2x \ln\left(1+\sin^2 x\right)dx$

Đặt $u = \ln\left(1+\sin^2 x\right)$ và $dv = \sin 2xdx$. Suy ra: $du = \frac{\sin 2x}{1+\sin^2 x}dx$ và $v = 1+\sin^2 x$

$$A = \frac{1}{2}\left[\left(1+\sin^2 x\right)\ln\left(1+\sin^2 x\right)\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin 2xdx\right] = \frac{1}{2}\left[\left(1+\sin^2 x\right)\ln\left(1+\sin^2 x\right)\Big|_0^{\frac{\pi}{2}} - \left(\sin^2 x\right)\Big|_0^{\frac{\pi}{2}}\right] = \frac{\ln 4 - 1}{2}$$

13. $\int_1^e \frac{\ln x - 2}{x\ln x + x}dx$

Ta có: I = $\int_1^e \frac{\ln x - 2}{x\ln x + x}dx$ = $\int_1^e \frac{\ln x - 2}{(\ln x + 1)x}dx$

Đặt t = lnx + 1 $\Rightarrow$ dt = $\frac{1}{x}dx$; Đổi cận: x = 1 thì t = 1; x = e thì t = 2

Suy ra: I = $\int_1^2 \frac{t-3}{t}dt = \int_1^2 \left(1 - \frac{3}{t}\right)dt$ = $\left(t - \ln|t|\right)\Big|_1^2$ = 1 – ln2

14. $I = \int_{\sqrt{2}}^e x^3 \ln\frac{x^2-1}{x^2+1}dx$

Đặt $\begin{cases} u = \ln\frac{x^2-1}{x^2+1} \\ dv = x^3dx \end{cases}$ ta có $\begin{cases} du = \frac{4x}{x^4-1}dx \\ v = \frac{x^4-1}{4} \end{cases}$

I = $\frac{x^4-1}{4}\ln\frac{x^2-1}{x^2+1}\Big|_{\sqrt{2}}^e - \int_{\sqrt{2}}^e xdx = \frac{e^4-1}{4}\ln\frac{e^2-1}{e^2+1} - \frac{x^2}{2}\Big|_{\sqrt{2}}^e = \frac{e^4-1}{4}\ln\frac{e^2-1}{e^2+1} + \frac{3}{4}\ln 3 - \frac{e^2}{2} + 1$

15. $\int_1^e \frac{x^2\ln x + x\ln^2 x + x + 1}{x^2 + x\ln x}dx$

$$I = \int_1^e \ln xdx + \int_1^e \frac{1+\frac{1}{x}}{x+\ln x}dx = x\left(\ln x - 1\right)\Big|_1^e + \int_1^e \frac{d\left(x+\ln x\right)}{x+\ln x}$$

.

$$= \left[x\left(\ln x - 1\right) + \ln\left(x + \ln x\right)\right]\Big|_1^e = \ln\left[e\left(e+1\right)\right].$$

16. $I = \int\limits_0^1 x\ln\left(x^2+x+1\right)dx$

Đặt $\begin{cases} u = \ln\left(x^2+x+1\right) \\ dv = xdx \end{cases} \Rightarrow \begin{cases} du = \dfrac{2x+1}{x^2+x+1}dx \\ v = \dfrac{x^2}{2} \end{cases}$

$I = \dfrac{x^2}{2}\ln\left(x^2+x+1\right)|_0^1 - \dfrac{1}{2}\int\limits_0^1 \dfrac{2x^3+x^2}{x^2+x+1}dx = \dfrac{3}{4}\ln 3 - \dfrac{3}{4}J$ với

$J = \int\limits_0^1 \dfrac{dx}{\left(x+\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2}$. Đặt $x + \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\tan t, t \in \left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right) \Rightarrow J = \dfrac{2\sqrt{3}}{2}\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} dt = \dfrac{\pi\sqrt{3}}{9}$.

Vậy I = $\dfrac{3}{4}\ln 3 - \dfrac{\pi\sqrt{3}}{12}$

17. $\int\limits_{\frac{1}{2}}^{2} (x+1-\dfrac{1}{x})e^{x+\frac{1}{x}}dx$

:I = $\int\limits_{\frac{1}{2}}^{2} (x+1-\dfrac{1}{x})e^{x+\frac{1}{x}}dx = \int\limits_{\frac{1}{2}}^{2} e^{x+\frac{1}{x}}dx + \int (x-\dfrac{1}{x})e^{x+\frac{1}{x}}dx = I_1 + I_2$.

Tính I_1 theo phương pháp từng phần I_1 = $xe^{x+\frac{1}{x}}\Big|_{\frac{1}{2}}^{2} - \int\limits_{\frac{1}{2}}^{2} (x-\dfrac{1}{x})e^{x+\frac{1}{x}}dx = \dfrac{3}{2}e^{\frac{5}{2}} - I_2 \Rightarrow I = \dfrac{3}{2}e^{\frac{5}{2}}$.

HT 2.Tính các tích phân sau:

1. $A=\int\limits_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{dx}{x\sqrt{1-x^2}}$	**7.** $\int\limits_{\frac{1}{3}}^{1}\frac{x}{3x+\sqrt{9x^2-1}}dx$
2. $I=\int\limits_0^1 x^3\ln\left(\frac{4-x^2}{4+x^2}\right)dx$	**8.** $I=\int\limits_{\sqrt{3}}^{2\sqrt{2}}\frac{x^4}{\left(x-\frac{1}{x}\right)\sqrt{x^2+1}}dx$
3. $\int\limits_0^1\frac{dx}{1+\sqrt{1-x^2}}$	**9.** $I=\int\limits_1^5\frac{x^2+1}{x\sqrt{3x+1}}dx$
4. $I=\int\limits_2^6\frac{dx}{2x+1+\sqrt{4x+1}}$	**10.** $\int\limits_0^1\frac{x(e^x+1)}{(x+1)^2}dx$
5. $I=\int\limits_0^1\frac{x^3+3x}{x^4-5x^2+6}dx$	**11.** $\int\limits_1^2\frac{1-x^5}{x\left(1+x^5\right)^2}dx$
6. $I=\int\limits_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}\frac{3x^4+x^2+1}{x^2\sqrt[3]{x^3+x}}dx$	**12.** $\int\limits_0^1(x^2e^{x^3}+\frac{\sqrt[4]{x}}{1+\sqrt{x}})dx$

Bài giải

1. $A=\int\limits_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{dx}{x\sqrt{1-x^2}}$

Đặt $t=\sqrt{1-x^2}\Rightarrow t^2=1-x^2\Rightarrow 2tdt=-2xdx\Rightarrow\frac{dx}{x}=-\frac{tdt}{x^2}$

$\Rightarrow\frac{dx}{x}=-\frac{tdt}{1-t^2}=\frac{tdt}{t^2-1}$

+ Đổi cận: $x=\frac{1}{2}\Rightarrow t=\frac{\sqrt{3}}{2};x=\frac{\sqrt{3}}{2}\Rightarrow t=\frac{1}{2}$

$$A=\int\limits_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}\frac{dt}{t^2-1}=\int\limits_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{dt}{1-t^2}=\frac{1}{2}\ln\left|\frac{t+1}{1-t}\right|\Bigg|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}=\frac{1}{2}\ln\left(\frac{7+4\sqrt{3}}{3}\right)$$

2. $I=\int\limits_0^1 x^3\ln\left(\frac{4-x^2}{4+x^2}\right)dx$

Đặt $\begin{cases} u = \ln\left(\dfrac{4-x^2}{4+x^2}\right) \\ dv = x^3dx \end{cases} \Rightarrow \begin{cases} du = \dfrac{16x}{x^4-16}dx \\ v = \dfrac{x^4-16}{4} \end{cases}$

Do đó $I = \dfrac{1}{4}\left(x^4-16\right)\ln\left(\dfrac{4-x^2}{4+x^2}\right)\Bigg|_0^1 - 4\int_0^1 xdx = -\dfrac{15}{4}\ln\left(\dfrac{3}{5}\right) - 2$

3. $\int_0^1 \dfrac{dx}{1+\sqrt{1-x^2}}$

Đặt x = sint với t $\in [-\dfrac{\pi}{2};\dfrac{\pi}{2}]$. Ta có: $dx = \cos tdt$ và $\sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t}$ =|cost| = cost.

Đổi cận: Với x =0 thì t = 0; Với x = 1 thì t = $\dfrac{\pi}{2}$. Từ đó:

$$\int_0^1 \frac{dx}{1+\sqrt{1-x^2}} = \int_0^{\pi/2} \frac{\cos tdt}{1+\cos t} = \int_0^{\pi/2} \frac{2\cos s^2(t/2)-1}{2\cos s^2(t/2)}dt$$

$= \int_0^{\pi/2} dt - \int_0^{\pi/2} \dfrac{d(t/2)}{\cos^2(t/2)}$ =(t – tan (t/2)) $|_0^{\pi/2}$ = $\dfrac{\pi}{2}$ -1

4. $I = \int_2^6 \dfrac{dx}{2x+1+\sqrt{4x+1}}$

Đặt $t = \sqrt{4x+1} \Rightarrow x = \dfrac{t^2-1}{4} \Rightarrow dx = \dfrac{tdt}{2}$ $\quad t\left(2\right) = 3, t\left(6\right) = 5$

Khi đó $I = \int_3^5 \dfrac{tdt}{\left(t+1\right)^2} = \int_3^5 \left(\dfrac{1}{t+1} - \dfrac{1}{\left(t+1\right)^2}\right)dt = \left(\ln\left|t+1\right| + \dfrac{1}{t+1}\right)\Big|_3^5 = \ln\dfrac{3}{2} - \dfrac{1}{12}$

$= \left(\ln\left|t+1\right| + \dfrac{1}{t+1}\right)\Big|_3^5 = \ln\dfrac{3}{2} - \dfrac{1}{12}$

5. $I = \int_0^1 \dfrac{x^3+3x}{x^4-5x^2+6}dx$

$$I = \frac{1}{2}\int_0^1 \frac{x^2+3}{(x^2-2)(x^2-3)}dx^2 = \frac{1}{2}\int_0^1 \frac{x^2-2+5}{(x^2-2)(x^2-3)}dx^2$$

$$=\frac{1}{2}\int_0^1\frac{dx^2}{x^2-3}+\frac{5}{2}\int_0^1\left(\frac{1}{x^2-3}-\frac{1}{x^2-2}\right)dx^2=\left(\frac{1}{2}\ln\left|x^2-3\right|+\frac{5}{2}\ln\left|\frac{x^2-3}{x^2-2}\right|\right)\Big|_0^1$$

$$=\left(\frac{1}{2}\ln 2+\frac{5}{2}\ln 2\right)-\left(\frac{1}{2}\ln 3+\frac{5}{2}\ln\frac{3}{2}\right)=3\ln 2-3\ln 3+\frac{5}{2}\ln 2=3\ln\frac{2}{3}+\frac{5}{2}\ln 2$$

6. $I=\int_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}\frac{3x^4+x^2+1}{x^2\sqrt[3]{x^3+x}}dx=\int_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}\frac{3x^4+x^2}{x^2\sqrt[3]{x^3+x}}dx+\int_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}\frac{1}{x^2\sqrt[3]{x^3+x}}dx=I_1+I_2$

$*$ *Tính* $I_2=\int_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}\frac{1}{\sqrt[3]{1+\frac{1}{x^2}}}\frac{1}{x^3}dx=-\frac{1}{2}\int_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}\frac{1}{\sqrt[3]{1+\frac{1}{x^2}}}d\left(1+\frac{1}{x^2}\right)=-\frac{3}{4}\sqrt[3]{\left(1+\frac{1}{x^2}\right)^2}\Bigg|_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}}=\frac{15}{4}$

Vậy: $I=\frac{322}{91}$.

7. $\int_{\frac{1}{3}}^{1}\frac{x}{3x+\sqrt{9x^2-1}}dx$

$$I=\int_{\frac{1}{3}}^{1}\frac{x}{3x+\sqrt{9x^2-1}}dx=\int_{\frac{1}{3}}^{1}x(3x-\sqrt{9x^2-1})dx=\int_{\frac{1}{3}}^{1}3x^2dx-\int_{\frac{1}{3}}^{1}x\sqrt{9x^2-1}dx$$

$$I_1=\int_{\frac{1}{3}}^{1}3x^2dx=x^3\Big|_{\frac{1}{3}}^{1}=\frac{26}{27}$$

$$I_2=\int_{\frac{1}{3}}^{1}x\sqrt{9x^2-1}dx=\frac{1}{18}\int_{\frac{1}{3}}^{1}\sqrt{9x^2-1}d(9x^2-1)=\frac{1}{27}(9x^2-1)^{\frac{3}{2}}\Big|_{\frac{1}{3}}^{1}=\frac{16\sqrt{2}}{27}.$$

Vậy $I=\frac{26-16\sqrt{2}}{27}$

8. $I=\int_{\sqrt{3}}^{2\sqrt{2}}\frac{x^4}{\left(x-\frac{1}{x}\right)\sqrt{x^2+1}}dx$

Ta có: $I=\int_{\sqrt{3}}^{2\sqrt{2}}\frac{x^5}{\left(x^2-1\right)\sqrt{x^2+1}}dx$

Đặt t = $\sqrt{x^2+1}$, suy ra $dt=\frac{x}{\sqrt{x^2+1}}dx$ & $x^2=t^2-1$ Đổi cận: $x=\sqrt{3}\Rightarrow t=2; x=2\sqrt{2}\Rightarrow t=3$

Khi đó $I=\int_2^3\frac{\left(t^2-1\right)^2}{t^2-2}dt$ Ta có I = $\int_2^3\frac{t^4-2t^2+1}{t^2-2}dt=\int_2^3 t^2dt+\int_2^3\frac{1}{t^2-2}dt$ = $\frac{1}{3}t^3\Big|_2^3+\frac{1}{2\sqrt{2}}\int_2^3\left(\frac{1}{t-\sqrt{2}}-\frac{1}{t+\sqrt{2}}\right)dt$

= $\frac{19}{3}+\frac{1}{2\sqrt{2}}\left[\ln\left|t-\sqrt{2}\right|-\ln\left|t+\sqrt{2}\right|\right]_2^3$ = $\frac{19}{3}+\frac{\sqrt{2}}{4}\ln\left(\frac{4+\sqrt{2}}{4-\sqrt{2}}\right)$

9. $I=\int_1^5\frac{x^2+1}{x\sqrt{3x+1}}dx$

Đặt $t=\sqrt{3x+1}\ \Rightarrow dt=\frac{3dx}{2\sqrt{3x+1}}\Rightarrow dx=\frac{2tdt}{3}$.

Khi $x=1$ thì t = 2, và khi x = 5 thì t = 4.

Suy ra $I=\int_2^4\frac{\left(\frac{t^2-1}{3}\right)^2+1}{\frac{t^2-1}{3}.t}.\frac{2tdt}{3}\quad=\frac{2}{9}\int_2^4(t^2-1)dt+2\int_2^4\frac{dt}{t^2-1}=\frac{2}{9}\left(\frac{1}{3}t^3-t\right)\Bigg|_2^4+\ln\left|\frac{t-1}{t+1}\right|\Bigg|_2^4\quad=\frac{100}{27}+\ln\frac{9}{5}.$

10. $\int_0^1\frac{x(e^x+1)}{(x+1)^2}dx$

Đặt $\begin{cases}u=x(e^x+1)\\ dv=\frac{dx}{(x+1)^2}\end{cases}\Rightarrow\begin{cases}du=(e^x(x+1)+1)dx\\ v=\frac{-1}{x+1}\end{cases}$

I=$-\frac{x(e^x+1)}{x+1}\Big|_0^1+\int_0^1(e^x+\frac{1}{x+1})dx=-\frac{e+1}{2}+(e^x+\ln\left|x+1\right|)\Big|_0^1=\frac{e}{2}+\ln 2-\frac{3}{2}$.

Vậy I = $\frac{e}{2}+\ln 2-\frac{3}{2}$

11. $\int_1^2\frac{1-x^5}{x\left(1+x^5\right)^2}dx$

$$\int_1^2\frac{1-x^5}{x\left(1+x^5\right)^2}dx=\int_1^2\frac{1+x^5-2x^5}{x\left(1+x^5\right)^2}dx=\int_1^2\frac{1}{x\left(1+x^5\right)}dx-\int_1^2\frac{2x^4}{\left(1+x^5\right)^2}dx=I_1-I_2$$

$$x=\frac{1}{t}\Rightarrow I_1=\int_1^{1/2}\frac{\left(-\frac{1}{t^2}\right)}{\frac{1}{t}\left(1+\frac{1}{t^5}\right)}dt=\int_{1/2}^{1}\frac{t^4}{t^5+1}dt=\frac{1}{5}\int_{1/2}^{1}\frac{1}{t^5+1}d\left(t^5+1\right)=\frac{1}{5}\ln\left|t^5+1\right|\Big|_{1/2}^{1}=\frac{1}{5}\left(6\ln 2-\ln 33\right)$$

$$I_2=\frac{2}{5}\int_1^2\frac{1}{\left(1+x^5\right)^2}d\left(x^5+1\right)=-\frac{2}{5}.\left(\frac{1}{x^5+1}\right)\Big|_1^2=\frac{31}{165}$$

$$I=\frac{1}{5}\left(6\ln 2-\ln 33\right)-\frac{31}{165}$$

12. $\int_0^1 (x^2e^{x^3}+\frac{\sqrt[4]{x}}{1+\sqrt{x}})dx$

Đặt I = $\int_0^1 (x^2e^{x^3}+\frac{\sqrt[4]{x}}{1+\sqrt{x}})dx$. Ta có I = $\int_0^1 x^2e^{x^3}dx+\int_0^1\frac{\sqrt[4]{x}}{1+\sqrt{x}}dx$

Ta tính $I_1=\int_0^1 x^2e^{x^3}dx$ Đặt t = x^3 ta có $I_1=\frac{1}{3}\int_0^1 e^t dt=\frac{1}{3}e^t\Big|_0^1=\frac{1}{3}e-\frac{1}{3}$

Ta tính $I_2=\int_0^1\frac{\sqrt[4]{x}}{1+\sqrt{x}}dx$ Đặt t = $\sqrt[4]{x}\Rightarrow x=t^4\Rightarrow dx=4t^3dt$

Khi đó $I_2=4\int_0^1\frac{t^4}{1+t^2}dx=4\int_0^1(t^2-1+\frac{1}{1+t^2})dt=4(-\frac{2}{3}+\frac{\pi}{4})$. Vậy I = I_1+ I_2 $=\frac{1}{3}e+\pi-3$

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HT 3.Tính các tích phân sau:

1. $\int\limits_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x).\sin 2x dx$

2. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\left(x + 2\sin x - 3\right)\cos x}{\sin^3 x} dx$

3. $M = \int\limits_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx$

4. $I = \int\limits_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x} dx$

5. $\int\limits_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3}\sin 2x + 2\cos^2 x} dx$

6. $I = \int\limits_0^{\frac{\pi}{4}} \frac{x\sin x + \sin 2x}{\cos^2 x} dx$

7. $M = \int\limits_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx$

8. $I = \int\limits_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos\left(2x - \frac{\pi}{4}\right)} dx$

9. $I = \int\limits_0^{\frac{\pi}{2}} \cos 2x\left(\sin^4 x + \cos^4 x\right) dx$

10. $I = \int\limits_0^{\frac{\pi}{4}} \frac{\tan x}{\left(4\cos x - \sin x\right)\cos x} dx$

11. $\int\limits_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x).\sin 2x dx$

12. $\int\limits_o^{\pi} \frac{x}{1 + \sin x} dx$

13. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\left(x + 2\sin x - 3\right)\cos x}{\sin^3 x} dx$

14. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot x\, dx}{1 + \sin^4 x}$

15. $\int\limits_0^{\frac{\pi}{3}} \frac{3\sin x - \sin 2x}{(\cos 2x - 3\cos x + 1)(3 - 2\sin^2 x)} dx$

16. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x.\sin\left(x + \frac{\pi}{4}\right)} dx$

17. $I = \int\limits_0^{\pi} x\left(\cos x + \sin^5 x\right) dx$

Bài giải

1.I= $\int\limits_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x).\sin 2x dx = 2\int\limits_0^{\frac{\pi}{2}} e^{\cos x}.\cos x.\sin x dx + \int\limits_0^{\frac{\pi}{2}} \sin x.\sin 2x dx$

$J = \int\limits_0^{\frac{\pi}{2}} e^{\cos x}.\cos x.\sin x dx$

Đặt t = cosx có J = $\int\limits_0^1 t.e^t.dt = t.e^t\Big|_0^1 - \int\limits_0^1 e^t.dt = 1$

$$K=\int_0^{\frac{\pi}{2}}\sin x.\sin 2x dx=\frac{1}{2}\int_0^{\frac{\pi}{2}}(\cos x-\cos 3x)dx=\frac{1}{2}(\sin x-\frac{1}{3}\sin 3x)\Big|_0^{\frac{\pi}{2}}=\frac{2}{3}$$

Vậy: I= $\int_0^{\frac{\pi}{2}}(e^{\cos x}+\sin x).\sin 2x dx=2+\frac{2}{3}=\frac{8}{3}$

2. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\left(x+2\sin x-3\right)\cos x}{\sin^3 x}dx$

$$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\left(x+2\sin x-3\right)\cos x}{\sin^3 x}dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{x\cos x}{\sin^3 x}dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\left(2\sin x-3\right)\cos x}{\sin^3 x}dx$$

$$I_1=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{x\cos x}{\sin^3 x}dx=-\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}xd\left(\frac{1}{\sin^2 x}\right)=-\frac{1}{2}\frac{x}{\sin^2 x}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}+\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1}{\sin^2 x}dx=-\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{2}\right)-\frac{1}{2}\cot x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}=\frac{1}{2}$$

$I_2=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\left(2\sin x-3\right)\cos x}{\sin^3 x}dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{2\sin x-3}{\sin^3 x}d\left(\sin x\right)=2\sqrt{2}-\frac{7}{2}$. Vậy $I=I_1+I_2=2\sqrt{2}-3$.

3. $M=\int_0^{\frac{\pi}{4}}\frac{\cos x+\sin 2x}{1+\cos 2x}dx$

$$M=\int_0^{\frac{\pi}{4}}\frac{\cos x+\sin 2x}{1+\cos 2x}dx=\underbrace{\int_0^{\frac{\pi}{4}}\frac{\sin 2x}{1+\cos 2x}dx}_{M_1}+\underbrace{\int_0^{\frac{\pi}{4}}\frac{\cos x}{1+\cos 2x}dx}_{M_2};$$

$$M_1=-\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{d\left(1+\cos 2x\right)}{1+\cos 2x}=-\frac{1}{2}\ln\left|1+\cos 2x\right|\Big|_0^{\frac{\pi}{4}}=\frac{1}{2}\ln 2,\ M_2=\int_0^{\frac{\pi}{4}}\frac{\cos x}{1+\cos 2x}dx=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{\cos x}{1-\sin^2 x}dx=$$

Đặt $u=\sin t$ $M_2=\frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}}\frac{du}{1-u^2}=\frac{1}{4}\int_0^{\frac{1}{\sqrt{2}}}\left(\frac{1}{1-u}+\frac{1}{1+u}\right)du=\frac{1}{4}\ln\left|\frac{u+1}{u-1}\right|\Big|_0^{\frac{1}{\sqrt{2}}}=\frac{1}{2}\ln(1+\sqrt{2})$

Vậy $M=\frac{1}{2}\ln(2+2\sqrt{2})$

4. $I = \int\limits_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x} dx$

$$I = \int\limits_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x} dx = -\int\limits_0^{\frac{\pi}{6}} \frac{\tan^2 x + 1}{(\tan x + 1)^2} dx$$

Đặt $t = \tan x \Rightarrow dt = \frac{1}{\cos^2 x} dx = (\tan^2 x + 1) dx$

Đổi cận: $x = 0 \Rightarrow t = 0, x = \frac{\pi}{6} \Rightarrow t = \frac{1}{\sqrt{3}}$

Suy ra $I = -\int\limits_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(t+1)^2} = \left.\frac{1}{t+1}\right|_0^{\frac{1}{\sqrt{3}}} = \frac{1-\sqrt{3}}{2}$

5. $\int\limits_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3}\sin 2x + 2\cos^2 x} dx$

$$I = \int\limits_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3}\sin 2x + 2\cos^2 x} dx = \int\limits_0^{\frac{\pi}{2}} \sqrt{(\sin x - \sqrt{3}\cos x)^2} dx = \int\limits_0^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx$$

$\sin x - \sqrt{3}\cos x = 0 \Leftrightarrow \tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} + k\pi$ Do $x \in \left(0; \frac{\pi}{2}\right)$ nên $x = \frac{\pi}{3}$

$$I = \int\limits_0^{\frac{\pi}{3}} \left|\sin x - \sqrt{3}\cos x\right| dx + \int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = \left|\int\limits_0^{\frac{\pi}{3}} (\sin x - \sqrt{3}\cos x) dx\right| + \left|\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sin x - \sqrt{3}\cos x) dx\right|$$

$$= \left|\left.\left(-\cos x - \sqrt{3}\sin x\right)\right|_0^{\frac{\pi}{3}}\right| + \left|\left.\left(-\cos x - \sqrt{3}\sin x\right)\right|_{\frac{\pi}{3}}^{\frac{\pi}{2}}\right| = \left|-\frac{1}{2} - \frac{3}{2} + 1\right| + \left|-\sqrt{3} + \frac{1}{2} + \frac{3}{2}\right| = 3 - \sqrt{3}$$

6. $I = \int\limits_0^{\frac{\pi}{4}} \frac{x\sin x + \sin 2x}{\cos^2 x} dx$

+ Ta có $I = \int\limits_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^2 x} dx + 2\int\limits_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx$ Đặt $I_1 = \int\limits_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^2 x} dx; I_2 = 2\int\limits_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx$

+Tính I_1: Đặt $u = x \Rightarrow du = dx; v = \int \frac{\sin x}{\cos^2 x}dx = -\int \cos^{-2} x d(\cos x) = \frac{1}{\cos x}$

$$\Rightarrow I_1 = \frac{x}{\cos x}\Bigg|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \frac{x}{\cos x}\Bigg|_0^{\frac{\pi}{4}} - \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right|\Bigg|_0^{\frac{\pi}{4}} = \frac{\pi\sqrt{2}}{4} - \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$$

+ Tính $I_2 = -2\int_0^{\frac{\pi}{4}} \frac{d(\cos x)}{\cos x} = -2\ln|\cos x|\Bigg|_0^{\frac{\pi}{4}} = -2\ln\frac{\sqrt{2}}{2}$

Vậy $I = I_1 + I_2 = \frac{\pi\sqrt{2}}{4} - \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}} - 2\ln\frac{\sqrt{2}}{2}$

7. $M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1+\cos 2x}dx$

$$M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1+\cos 2x}dx = \underbrace{\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1+\cos 2x}dx}_{M_1} + \underbrace{\int_0^{\frac{\pi}{4}} \frac{\cos x}{1+\cos 2x}dx}_{M_2}$$

$$M_1 = -\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{d\left(1+\cos 2x\right)}{1+\cos 2x} = -\frac{1}{2}\ln|1+\cos 2x|\Big|_0^{\frac{\pi}{4}} = \frac{1}{2}\ln 2 \quad M_2 = \int_0^{\frac{\pi}{4}} \frac{\cos x}{1+\cos 2x}dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\cos x}{1-\sin^2 x}dx =$$

Đặt $u = \sin t$ $\quad M_2 = \frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{du}{1-u^2} = \frac{1}{4}\int_0^{\frac{1}{\sqrt{2}}} \left(\frac{1}{1-u} + \frac{1}{1+u}\right)du = \frac{1}{4}\ln\left|\frac{u+1}{u-1}\right|\Big|_0^{\frac{1}{\sqrt{2}}} = \frac{1}{2}\ln(1+\sqrt{2})$

Vậy $M = \frac{1}{2}\ln(2+2\sqrt{2})$

8. $I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos\left(2x - \frac{\pi}{4}\right)}dx$

$$I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos\left(2x - \frac{\pi}{4}\right)}dx \quad I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos\left(2x - \frac{\pi}{4}\right)}dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\frac{\cos^2 x - \sin^2 x}{\sin x.\cos x}}{\sin 2x\left(\cos 2x.\cos\frac{\pi}{4} + \sin 2x.\sin\frac{\pi}{4}\right)}dx$$

$$= 2\sqrt{2}\int\limits_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot 2x}{\sin 2x\left(\cos 2x + \sin 2x\right)}dx = 2\sqrt{2}\int\limits_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot 2x}{1+\cot 2x}.\frac{1}{\sin^2 2x}dx$$

Đặt $t = \cot 2x \Rightarrow dt = -\frac{2}{\sin^2 2x}dx \Rightarrow -\frac{1}{2}dt = \frac{1}{\sin^2 2x}dx$. Đổi cận: $x = \frac{\pi}{8} \Rightarrow t = 1;\ x = \frac{\pi}{4} \Rightarrow t = 0$

$$I = 2\sqrt{2}\int\limits_1^0 \frac{t}{1+t}.\left(-\frac{1}{2}dt\right) = \sqrt{2}\int\limits_0^1 \frac{t}{1+t}dt = \sqrt{2}\int\limits_0^1 \left(1-\frac{1}{t+1}\right)dt = \sqrt{2}\left(t - \ln\left|t+1\right|\right)\Big|_0^1 = \sqrt{2}\left(1-\ln 2\right)$$

9. $I = \int\limits_0^{\frac{\pi}{2}} \cos 2x\left(\sin^4 x + \cos^4 x\right)dx$

$$I = \int\limits_0^{\frac{\pi}{2}} \cos 2x\left(1-\frac{1}{2}\sin^2 2x\right)dx = \frac{1}{2}\int\limits_0^{\frac{\pi}{2}}\left(1-\frac{1}{2}\sin^2 2x\right)d\left(\sin 2x\right)$$

$$= \frac{1}{2}\int\limits_0^{\frac{\pi}{2}} d\left(\sin 2x\right) - \frac{1}{4}\int\limits_0^{\frac{\pi}{2}} \sin^2 2x d\left(\sin 2x\right) = \frac{1}{2}\sin 2x\Big|_0^{\frac{\pi}{2}} - \frac{1}{12}\sin^3 2x\Big|_0^{\frac{\pi}{2}} = 0$$

10. $I = \int\limits_0^{\frac{\pi}{4}} \frac{\tan x}{\left(4\cos x - \sin x\right)\cos x}dx$

Ta có: $I = \int\limits_0^{\frac{\pi}{4}} \frac{\tan x}{\left(4-\tan x\right)\cos^2 x}dx$

Đặt: $\tan x - 4 = t \Rightarrow \frac{dx}{cos^2 x} = dt$. Đổi cận: Với $x = 0 \Rightarrow t = -4;\ \ x = \frac{\pi}{4} \Rightarrow t = -3$

Suy ra: $I = -\int\limits_{-4}^{-3} \frac{(t+4).dt}{t} = -\int\limits_{-4}^{-3}(1+\frac{4}{t})dt = -(t + 4\ln\left|t\right|)\Big|_{-4}^{-3} = 4\ln\frac{4}{3} - 1$

11. $\int\limits_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x).\sin 2x dx$

$$\int\limits_0^{\frac{\pi}{2}} (e^{\cos x} + \mathrm{s\,inx}).\sin 2x dx = 2\int\limits_0^{\frac{\pi}{2}} e^{\cos x}.\cos x.\sin x dx + \int\limits_0^{\frac{\pi}{2}} \mathrm{s\,inx}.\sin 2x dx$$

$$I=\int_0^{\frac{\pi}{2}} e^{\cos x}.\cos x.\sin x dx$$

Đặt t = cosx có I = $\int_0^1 t.e^t.dt = t.e^t\Big|_0^1 - \int_0^1 e^t.dt = 1$

$$K=\int_0^{\frac{\pi}{2}} \mathrm{s\,inx}.\sin 2x dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos x - cos3x).dx = \frac{1}{2}(\mathrm{s\,inx} - \frac{1}{3}\sin 3x)\Big|_0^{\frac{\pi}{2}} = \frac{2}{3}$$

$$\int_0^{\frac{\pi}{2}} (e^{\cos x} + \mathrm{s\,inx}).\sin 2x dx = 2 + \frac{2}{3} = \frac{8}{3}$$

12. $\int_o^{\pi} \frac{x}{1+\sin x}dx$

I = $\int_o^{\pi} \frac{x}{1+\sin x}dx$ = $\int_o^{\pi} \frac{x}{(\sin\frac{x}{2}+\cos\frac{x}{2})^2}dx$ = $\int_o^{\pi} \frac{x}{2\cos^2\left(\frac{x}{2}-\frac{\pi}{4}\right)}dx$

Đặt $\begin{cases} u = x \\ dv = \dfrac{dx}{2\cos^2\left(\frac{x}{2}-\frac{\pi}{4}\right)} \end{cases}$ => $\begin{cases} du = dx \\ v = \tan\left(\frac{x}{2}-\frac{\pi}{4}\right) \end{cases}$ => I = $x\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)\Big|_0^{\pi} - \int_0^{\pi} \tan\left(\frac{x}{2}-\frac{\pi}{4}\right)dx$

=> I = π + 2ln$\left|\cos\left(\frac{x}{2}-\frac{\pi}{4}\right)\right|\Big|_0^{\pi}$

13.. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x+2\sin x-3)\cos x}{\sin^3 x}dx$

$$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x+2\sin x-3)\cos x}{\sin^3 x}dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\cos x}{\sin^3 x}dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(2\sin x-3)\cos x}{\sin^3 x}dx$$

$$I_1=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\cos x}{\sin^3 x}dx = -\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} xd\left(\frac{1}{\sin^2 x}\right) = -\frac{1}{2}\frac{x}{\sin^2 x}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^2 x}dx = -\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{2}\right) - \frac{1}{2}\cot x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2}$$

$$I_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\left(2\sin x - 3\right)\cos x}{\sin^3 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{2\sin x - 3}{\sin^3 x} d\left(\sin x\right) = 2\sqrt{2} - \frac{7}{2}$$

Vậy $I = I_1 + I_2 = 2\sqrt{2} - 3$.

14. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot x\, dx}{1+\sin^4 x}$

Ta có: I = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot x}{1+\sin^4 x} dx$ = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x(1+\sin^4 x)} dx$ = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin^3 x \cos x}{\sin^4 x\left(1+\sin^4 x\right)} dx$.

Đặt $t = \sin^4 x$, ta có: $x = \frac{\pi}{4} \Rightarrow t = \frac{1}{4}$, $x = \frac{\pi}{2} \Rightarrow t = 1$ và $dt = 4\sin^3 x \cos x dx$.

Khi đó I = $\frac{1}{4}\int_{\frac{1}{4}}^{1} \frac{dt}{t\left(t+1\right)}$ = $\frac{1}{4}\int_{\frac{1}{4}}^{1} \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$ = $\frac{1}{4} \ln \frac{t}{t+1}\Big|_{\frac{1}{4}}^{1} = \frac{1}{4}\ln\frac{5}{2}$.

15. $\int_{0}^{\frac{\pi}{3}} \frac{3\sin x - \sin 2x}{(\cos 2x - 3\cos x + 1)(3 - 2\sin^2 x)} dx$

Ta có I = $\int_{0}^{\frac{\pi}{3}} \frac{3\sin x - \sin 2x}{(2\cos^2 x - 3\cos x)(3 - 2\sin^2 x)} dx$ = $\int_{0}^{\frac{\pi}{3}} \frac{\sin x(3 - 2\cos x)}{(2\cos x - 3).\cos x.\ (1 + 2\cos^2 x)} dx$

= $\int_{0}^{\frac{\pi}{3}} \frac{\sin x(3 - 2\cos x)}{(2\cos x - 3).\cos x.\ (1 + 2\cos^2 x)} dx$ = $\int_{0}^{\frac{\pi}{3}} \frac{-\sin x}{\cos x.\ (1 + 2\cos^2 x)} dx$

= $\int_{0}^{\frac{\pi}{3}} \frac{\cos x.(-\sin x).dx}{\cos^2 x.\ (1 + 2\cos^2 x)}$. Đặt t = $2\cos^2 x \Rightarrow dt = 4\cos x.(-\sin x)dx$

Đổi cận: Khi x = 0 $\Rightarrow t = 2$; khi $x = \frac{\pi}{3} \Rightarrow t = \frac{1}{2}$. Khi đó I = $\frac{1}{2}\int_{2}^{\frac{1}{2}} \frac{dt}{t.(1+t)}$ =

= $\frac{1}{2}\int_{2}^{\frac{1}{2}} \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$ = $\frac{1}{2}\ln\left|\frac{t}{t+1}\right|\Big|_{2}^{\frac{1}{2}}$ = $\frac{1}{2}.(\ln\frac{1}{3} - \ln\frac{2}{3})$ = $\frac{1}{2}.\ln\frac{1}{2}$. ***Vậy I =*** $-\frac{1}{2}\ln 2$.

16. $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sin x.\sin\left(x+\frac{\pi}{4}\right)}dx$

Tính $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sin x\sin\left(x+\frac{\pi}{4}\right)}dx=\sqrt{2}\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sin x\left(\sin x+\cos x\right)}dx=\sqrt{2}\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sin^2 x\left(1+\cot x\right)}dx$

Đặt 1+cotx=t $\Rightarrow \frac{1}{\sin^2 x}dx=-dt$ Khi $x=\frac{\pi}{6}\Leftrightarrow t=1+\sqrt{3};\quad x=\frac{\pi}{3}\Leftrightarrow t=\frac{\sqrt{3}+1}{\sqrt{3}}$

Vậy $I=\sqrt{2}\int\limits_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1}\frac{t-1}{t}dt=\sqrt{2}\left(t-\ln t\right)\Big|_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1}=\sqrt{2}\left(\frac{2}{\sqrt{3}}-\ln\sqrt{3}\right)$

17. $I=\int\limits_0^{\pi}x\left(\cos x+\sin^5 x\right)dx$

$I=\int\limits_0^{\pi}x\left(\cos x+\sin^5 x\right)dx$

* $I=\int\limits_0^{\pi}x\left(\cos x+\sin^5 x\right)dx=\underbrace{\int\limits_0^{\pi}x.\cos x.dx}_{I_1}+\underbrace{\int\limits_0^{\pi}x.\sin^5 x.dx}_{I_2}$.

* $I_1=\int\limits_0^{\pi}x.\cos x.dx=x.\sin x\Big|_0^{\pi}-\int\limits_0^{\pi}\sin x.dx=x.\sin x\Big|_0^{\pi}+\cos x\Big|_0^{\pi}=-2$

* Với I_2 ta đặt $x=\pi-t\Rightarrow I_2=\frac{\pi}{2}\int\limits_0^{\pi}\left(1-\cos^2 x\right)^2 d\left(-\cos x\right)=\frac{8\pi}{15}$. * Vậy $I=\frac{8\pi}{15}-2$

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HT 4.Tính các tích phân sau:

1. $I=\int\limits_0^{\ln 6}\frac{e^x}{3\sqrt{3+e^x}+2e^x+7}\mathrm{d}x.$	**3.** $I=\int\limits_0^1\frac{(x^2+x)e^x}{x+e^{-x}}dx$
2. $I=\int\limits_0^1\frac{(x-1)e^x+x+1}{1+e^x}dx$	**4.** $I=\int\limits_0^{\ln 2}\frac{x}{e^x+e^{-x}+2}\mathrm{d}x.$

Bài giải

1. $I=\int\limits_0^{\ln 6}\frac{e^x}{3\sqrt{3+e^x}+2e^x+7}\mathrm{d}x.$

Đặt $\sqrt{3+e^x}=t.$ Khi đó $e^x=t^2-3\Rightarrow e^xdx=2t\mathrm{d}t.$

Khi $x=0\Rightarrow t=2,$ khi $x=\ln 6\Rightarrow t=3.$

Suy ra $I=\int\limits_2^3\frac{2t\mathrm{d}t}{3t+2(t^2-3)+7}=2\int\limits_2^3\frac{t}{2t^2+3t+1}\mathrm{d}t$

$$=2\int\limits_2^3\frac{t}{(t+1)(2t+1)}\mathrm{d}t=2\int\limits_2^3\left(\frac{1}{t+1}-\frac{1}{2t+1}\right)\mathrm{d}t=2\ln\left|t+1\right|\Big|_2^3-\ln\left|2t+1\right|\Big|_2^3=(2\ln 4-2\ln 3)-(\ln 7-\ln 5)=\ln\frac{80}{63}.$$

2. $I=\int\limits_0^1\frac{(x-1)e^x+x+1}{1+e^x}dx$

$$I=\int\limits_0^1\frac{xe^x-e^x+x+1}{1+e^x}dx=\int\limits_0^1\frac{x(e^x+1)+(1+e^x)-2e^x}{1+e^x}dx=\int\limits_0^1(x+1)dx-2\int\limits_0^1\frac{e^x}{1+e^x}dx=I_1-2I_2$$

Tính $I_1=\int\limits_0^1(x+1)dx=\left(\frac{x^2}{2}+x\right)\Big|_0^1=\frac{3}{2}$ Tính $I_2=\int\limits_0^1\frac{e^x}{1+e^x}dx=\int\limits_0^1\frac{d(e^x+1)}{e^x+1}=\ln(e^x+1)\Big|_0^1=\ln\frac{e+1}{2}$

Vậy $I=\frac{3}{2}-2\ln\frac{e+1}{2}.$

3. $I=\int\limits_0^1\frac{(x^2+x)e^x}{x+e^{-x}}dx$

Ta có I= $\int\limits_0^1\frac{(x^2+x)e^x}{x+e^{-x}}dx$ = $\int\limits_0^1\frac{xe^x.(x+1)e^x}{xe^x+1}dx$

Đặt $t=x.e^x+1\Rightarrow dt=(x+1)e^xdx$ $x=0\Rightarrow t=1;\ x=1\Rightarrow t=e+1$

Suy ra I= $\int_0^1 \frac{xe^x.(x+1)e^x}{xe^x+1}dx = \int_1^{e+1}\frac{(t-1)}{t}dt = \int_1^{e+1}\left(1-\frac{1}{t}\right)dt$. Vậy I $= \left(t-\ln|t|\right)\Big|_1^{e+1} = e-\ln(e+1)$.

4. $I = \int_0^{\ln 2}\frac{x}{e^x+e^{-x}+2}\mathrm{d}x.$

Ta có $I = \int_0^{\ln 2}\frac{xe^x}{(e^x+1)^2}\mathrm{d}x.$

Đặt $u = x \Rightarrow \mathrm{d}u = \mathrm{d}x,\ \mathrm{d}v = \frac{e^x}{(e^x+1)^2}\mathrm{d}x \Rightarrow v = -\frac{1}{e^x+1}.$

Theo công thức tích phân từng phần ta có: $I = -\frac{x}{e^x+1}\Big|_0^{\ln 2} + \int_0^{\ln 2}\frac{\mathrm{d}x}{e^x+1} = -\frac{\ln 2}{3} + \int_0^{\ln 2}\frac{\mathrm{d}x}{e^x+1}$ (1)

Tính $I_1 = \int_0^{\ln 2}\frac{\mathrm{d}x}{e^x+1}$. Đặt $e^x = t$ ta có $x=0 \Rightarrow t=1$; $x=\ln 2 \Rightarrow t=2$ và $\mathrm{d}x = \frac{\mathrm{d}t}{t}$.

Suy ra $I_1 = \int_1^2\frac{\mathrm{d}t}{t(t+1)} = \int_1^2\left(\frac{1}{t}-\frac{1}{t+1}\right)\mathrm{d}t = \ln t\Big|_1^2 - \ln(t+1)\Big|_1^2 = 2\ln 2-\ln 3.$

Thay vào (1) ta được $I = \frac{5}{3}\ln 2-\ln 3.$

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PHẦN VIII TÍCH PHÂN HÀM TRỊ TUYỆT ĐỐI

ỨNG DỤNG TÍCH PHÂN

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HT 1.Tính các tích phân sau:

1. $I = \int\limits_{-3}^{2} \left|x^2 - 3x + 2\right| dx$.

2. $I = \int\limits_{0}^{\frac{\pi}{2}} \sqrt{5 - 4\cos^2 x - 4\sin x dx}$

3. $I = \int\limits_{-1}^{2} \left(\left|x\right| - \left|x - 1\right|\right) dx$

Bài giải

1. $I = \int\limits_{-3}^{2} \left|x^2 - 3x + 2\right| dx$

Bảng xét dấu

x	-3		1		2
$x^2 - 3x + 2$		$+$	0	$-$	0

$$I = \int\limits_{-3}^{1} \left(x^2 - 3x + 2\right) dx - \int\limits_{1}^{2} \left(x^2 - 3x + 2\right) dx = \frac{59}{2}.$$

2. $I = \int\limits_{0}^{\frac{\pi}{2}} \sqrt{5 - 4\cos^2 x - 4\sin x dx}$.

$$I = \int\limits_{0}^{\frac{\pi}{2}} \sqrt{4\sin^2 x - 4\sin x + 1 dx} = \int\limits_{0}^{\frac{\pi}{2}} \left|2\sin x - 1\right| dx.$$

Bảng xét dấu

x	0		$\frac{\pi}{6}$		$\frac{\pi}{2}$
$2\sin x - 1$		$-$	0	$+$	

$$I = -\int\limits_{0}^{\frac{\pi}{6}} \left(2\sin x - 1\right) dx + \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(2\sin x - 1\right) dx = 2\sqrt{3} - 2 - \frac{\pi}{6}.$$

3. $I = \int\limits_{-1}^{2} \left(\left|x\right| - \left|x-1\right|\right) dx$.

Cách 1.

$$I = \int\limits_{-1}^{2} \left(\left|x\right| - \left|x-1\right|\right) dx = \int\limits_{-1}^{2} \left|x\right| dx - \int\limits_{-1}^{2} \left|x-1\right| dx$$

$$= -\int\limits_{-1}^{0} xdx + \int\limits_{0}^{2} xdx + \int\limits_{-1}^{1} (x-1)dx - \int\limits_{1}^{2} (x-1)dx$$

$$= -\left.\frac{x^2}{2}\right|_{-1}^{0} + \left.\frac{x^2}{2}\right|_{0}^{2} + \left.\left(\frac{x^2}{2} - x\right)\right|_{-1}^{1} - \left.\left(\frac{x^2}{2} - x\right)\right|_{1}^{2} = 0.$$

Cách 2.

Bảng xét dấu

x	−1		0		1		2
x		–	0	+		+	
x – 1		–		–	0	+	

$$I = \int\limits_{-1}^{0} \left(-x + x - 1\right) dx + \int\limits_{0}^{1} \left(x + x - 1\right) dx + \int\limits_{1}^{2} \left(x - x + 1\right) dx$$

$$= -x\Big|_{-1}^{0} + \left(x^2 - x\right)\Big|_{0}^{1} + x\Big|_{1}^{2} = 0.$$

Vậy $I = 0$.

HT 2.Tính diện tích hình phẳng giới hạn bởi $y = \ln x,\ x = 1,\ x = e$ **và Ox.**

Bài giải

Do $\ln x \ge 0\ \forall x \in \left[1;\ e\right]$ nên:

$$S = \int\limits_{1}^{e} \left|\ln x\right| dx = \int\limits_{1}^{e} \ln x dx = x\left(\ln x - 1\right)\Big|_{1}^{e} = 1.$$

Vậy $S = 1$ (đvdt).

HT 3.Tính diện tích hình phẳng giới hạn bởi $y = -x^2 + 4x - 3,\ x = 0,\ x = 3$ **và** Ox

Bài giải

Bảng xét dấu

x	0		1		3
y		–	0	+	0

$$S = -\int_0^1 \left(-x^2 + 4x - 3\right)dx + \int_1^3 \left(-x^2 + 4x - 3\right)dx$$

$$= -\left(-\frac{x^3}{3} + 2x^2 + 3x\right)\Bigg|_0^1 + \left(-\frac{x^3}{3} + 2x^2 + 3x\right)\Bigg|_1^3 = \frac{8}{3}.$$

Vậy $S = \frac{8}{3}$ (đvdt).

HT 4.Tính diện tích hình phẳng giới hạn bởi các đường: $y = x^3 + 11x - 6,\ y = 6x^2$, $x = 0,\ x = 2$

Bài giải

Đặt $h(x) = (x^3 + 11x - 6) - 6x^2 = x^3 - 6x^2 + 11x - 6$

$h(x) = 0 \Leftrightarrow x = 1 \vee x = 2 \vee x = 3$ (loại).

Bảng xét dấu

x	0		1		2
h(x)		–	0	+	0

$$S = -\int_0^1 \left(x^3 - 6x^2 + 11x - 6\right)dx + \int_1^2 \left(x^3 - 6x^2 + 11x - 6\right)dx$$

$$= -\left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right)\Bigg|_0^1 + \left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right)\Bigg|_1^2 = \frac{5}{2}.$$

Vậy $S = \frac{5}{2}$ (đvdt).

HT 5.Tính diện tích hình phẳng giới hạn bởi các đường: $y = x^3 + 11x - 6,\ y = 6x^2$

Bài giải

Đặt $h(x) = (x^3 + 11x - 6) - 6x^2 = x^3 - 6x^2 + 11x - 6$

$h(x) = 0 \Leftrightarrow x = 1 \vee x = 2 \vee x = 3$.

Bảng xét dấu

x	1		2		3
h(x)	0	+	0	–	0

$$S = \int_1^2 \left(x^3 - 6x^2 + 11x - 6\right)dx - \int_2^3 \left(x^3 - 6x^2 + 11x - 6\right)dx$$

$$= \left.\left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right)\right|_1^2 - \left.\left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right)\right|_2^3 = \frac{1}{2}.$$

Vậy $S = \frac{1}{2}$ (đvdt).

HT 6.Tính diện tích hình phẳng giới hạn bởi $y = x^3,\ y = 4x$.

Bài giải

Phương trình hoành độ giao điểm:

$$x^3 = 4x \Leftrightarrow x = -2 \vee x = 0 \vee x = 2$$

$$\Rightarrow S = \left|\int_{-2}^0 \left(x^3 - 4x\right)dx\right| + \left|\int_0^2 \left(x^3 - 4x\right)dx\right| = \left|\left.\left(\frac{x^4}{4} - 2x^2\right)\right|_{-2}^0\right| + \left|\left.\left(\frac{x^4}{4} - 2x^2\right)\right|_0^2\right| = 8 .$$

Vậy $S = 8$ (đvdt).

HT 7.Tính diện tích hình phẳng giới hạn bởi $y = x^2 - 4|x| + 3$ và trục hoành.

Bài giải

Phương trình hoành độ giao điểm:

$$x^2 - 4|x| + 3 = 0 \Leftrightarrow t^2 - 4t + 3 = 0,\ t = |x| \ge 0 \Leftrightarrow \left[\begin{array}{l} t = 1 \\ t = 3 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} |x| = 1 \\ |x| = 3 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \pm 1 \\ x = \pm 3 \end{array}\right.$$

$$\Rightarrow S = \int_{-3}^3 \left|x^2 - 4|x| + 3\right|dx = 2\int_0^3 \left|x^2 - 4x + 3\right|dx$$

$$= 2\left[\left|\int_0^1 \left(x^2 - 4x + 3\right)dx\right| + \left|\int_1^3 \left(x^2 - 4x + 3\right)dx\right|\right]$$

$$= 2\left[\left|\left(\frac{x^3}{3} - 2x^2 + 3x\right)\Bigg|_0^1\right| + \left|\left(\frac{x^3}{3} - 2x^2 + 3x\right)\Bigg|_1^3\right|\right] = \frac{16}{3}.$$

Vậy $S = \dfrac{16}{3}$ (đvdt).

HT 8.Tính diện tích hình phẳng giới hạn bởi $y = \left|x^2 - 4x + 3\right|$ **và** $y = x + 3$.

Bài giải

Phương trình hoành độ giao điểm:

$$\left|x^2 - 4x + 3\right| = x + 3 \Leftrightarrow \begin{cases} x + 3 \geq 0 \\ \left[\begin{array}{l} x^2 - 4x + 3 = x + 3 \\ x^2 - 4x + 3 = -x - 3 \end{array}\right. \end{cases} \Leftrightarrow \left[\begin{array}{l} x = 0 \\ x = 5 \end{array}\right..$$

Bảng xét dấu

x	0 1 3 5
$x^2 - 4x + 3$	+ 0 – 0 +

$$\Rightarrow S = \left|\int_0^1 \left(x^2 - 5x\right)dx + \int_1^3 \left(-x^2 + 3x - 6\right)dx + \int_3^5 \left(x^2 - 5x\right)dx\right|$$

$$= \left|\left(\frac{x^3}{3} - \frac{5x^2}{2}\right)\Bigg|_0^1 + \left(\frac{-x^3}{3} + \frac{3x^2}{2} - 6x\right)\Bigg|_1^3 + \left(\frac{x^3}{3} - \frac{5x^2}{2}\right)\Bigg|_3^5\right| = \frac{109}{6}.$$

Vậy $S = \dfrac{109}{6}$ (đvdt).

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HT 9.Tính diện tích hình phẳng giới hạn bởi $y = \left|x^2 - 1\right|,\ \ y = \left|x\right| + 5$.

Bài giải

Phương trình hoành độ giao điểm:

$$\left|x^2 - 1\right| = \left|x\right| + 5 \Leftrightarrow \left|t^2 - 1\right| = t + 5,\ t = \left|x\right| \geq 0$$

$$\Leftrightarrow \begin{cases} t = \left|x\right| \geq 0 \\ \left[\begin{array}{l} t^2 - 1 = t + 5 \\ t^2 - 1 = -t - 5 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} t = \left|x\right| \geq 0 \\ t = 3 \end{cases} \Leftrightarrow x = \pm 3$$

$$\Rightarrow S=\int_{-3}^{3}\left|\left|x^2-1\right|-\left(\left|x\right|+5\right)\right|dx=2\int_{0}^{3}\left|\left|x^2-1\right|-\left(x+5\right)\right|dx$$

Bảng xét dấu

x	0		1		3
x^2-1		–	0	+	

$$\Rightarrow S=2\left|\int_{0}^{1}\left(-x^2-x-4\right)dx+\int_{1}^{3}\left(x^2-x-6\right)dx\right|$$

$$=2\left|\left(\frac{-x^3}{3}-\frac{x^2}{2}-4x\right)\Bigg|_0^1+\left(\frac{x^3}{3}-\frac{x^2}{2}-6x\right)\Bigg|_1^3\right|=\frac{73}{3}.$$

Vậy $S=\frac{73}{3}$ (đvdt).

HT 10.Tính diện tích hình phẳng giới hạn bởi $y=x,\ y=0,\ y=\sqrt{2-x^2}$ **.**

Bài giải

Ta có: $y=\sqrt{2-x^2}\Leftrightarrow x=\sqrt{2-y^2},\ x\ge 0$.

Phương trình tung độ giao điểm: $y=\sqrt{2-y^2}\Leftrightarrow y=1$.

$$\Rightarrow S=\int_{0}^{1}\left|\sqrt{2-y^2}-y\right|dy=\left|\int_{0}^{1}\left(\sqrt{2-y^2}-y\right)dy\right|$$

$$=\left|\int_{0}^{\frac{\pi}{4}}2\cos^2 tdt-\int_{0}^{1}ydy\right|=\left|\left(t+\frac{1}{2}\sin 2t\right)\Bigg|_0^{\frac{\pi}{4}}-\frac{y^2}{2}\Bigg|_0^1\right|.$$

Vậy $S=\frac{\pi}{4}$ (đvdt).

HT 11.Tính thể tích hình cầu do hình tròn $(C):x^2+y^2=R^2$ **quay quanh Ox.**

Bài Giải

Hoành độ giao điểm của (C) và Ox là $x^2=R^2\Leftrightarrow x=\pm R$.

Phương trình $(C):x^2+y^2=R^2\Leftrightarrow y^2=R^2-x^2$

$$\Rightarrow V = \pi\int_{-R}^{R}\left(R^2 - x^2\right)dx = 2\pi\int_{0}^{R}\left(R^2 - x^2\right)dx = 2\pi\left(R^2x - \frac{x^3}{3}\right)\Bigg|_0^R = \frac{4\pi R^3}{3}.$$

Vậy $V = \frac{4\pi R^3}{3}$ (đvtt).

HT 12.Tính thể tích hình khối do ellipse $(E): \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ quay quanh Oy.

Bài Giải

Tung độ giao điểm của (E) và Oy là $\frac{y^2}{b^2} = 1 \Leftrightarrow y = \pm b$.

Phương trình $(E): \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Leftrightarrow x^2 = a^2 - \frac{a^2y^2}{b^2}$

$$\Rightarrow V = \pi\int_{-b}^{b}\left(a^2 - \frac{a^2y^2}{b^2}\right)dy = 2\pi\int_{0}^{b}\left(a^2 - \frac{a^2y^2}{b^2}\right)dy = 2\pi\left(a^2y - \frac{a^2y^3}{3b^2}\right)\Bigg|_0^R = \frac{4\pi a^2 b}{3}.$$

Vậy $V = \frac{4\pi a^2 b}{3}$ (đvtt).

HT 13.Tính thể tích hình khối do hình phẳng giới hạn bởi các đường $y = x^2, y^2 = x$ quay quanh Ox.

Bài Giải

Hoành độ giao điểm: $\begin{cases} x \ge 0 \\ x^4 = x \end{cases} \Leftrightarrow \left[\begin{array}{l} x = 0 \\ x = 1 \end{array}\right.$.

$$\Rightarrow V = \pi\int_{0}^{1}\left|x^4 - x\right|dx = \pi\left|\int_{0}^{1}\left(x^4 - x\right)dx\right| = \pi\left|\left(\frac{1}{5}x^5 - \frac{1}{2}x^2\right)\Bigg|_0^1\right| = \frac{3\pi}{10}.$$

Vậy $V = \frac{3\pi}{10}$ (đvtt).

HT 14.Tính thể tích hình khối do hình phẳng giới hạn bởi các đường $x = -y^2 + 5$, $x = 3 - y$ quay quanh Oy.

Bài Giải

Tung độ giao điểm: $-y^2 + 5 = 3 - y \Leftrightarrow \left[\begin{array}{l} y = -1 \\ y = 2 \end{array}\right.$.

$$\Rightarrow V = \pi\int_{-1}^{2}\left|\left(-y^2 + 5\right)^2 - \left(3 - y\right)^2\right|dy = \pi\left|\int_{-1}^{2}\left(y^4 - 11y^2 + 6y + 16\right)dy\right|$$

$$= \pi \left| \left(\frac{y^5}{5} - \frac{11y^3}{3} + 3y^2 + 16y \right) \Big|_{-1}^{2} \right| = \frac{153\pi}{5}.$$

Vậy $V = \dfrac{153\pi}{5}$ (đvtt).

HT 15.Tính diện tích hình phẳng giới hạn bởi các đường có phương trình sau

1) $y = \sin x,\ y = 0,\ x = 0,\ x = 2\pi$ 2) $y = x^3,\ y = 0,\ x = -1,\ x = 2$ 3) $y = x^2 - 2x,\ y = -x^2 + 4x$ 4) $y = x^3,\ y = 4x,\ x = -1,\ x = 2$ 5) $y = -x^2 - 5,\ y = -6x,\ x = 0,\ x = 1$ 6) $y = -x^2 - 2,\ y = -3x,\ x = 0,\ x = 2$ 7) $y = -x^2 - 2x,\ y = -x - 2$ 8) $y = x^3 - 2x^2 - x + 2$ và trục hoành 9) $y = \left|x\right|^3 - 2x^2 - \left|x\right| + 2$ và trục hoành 10) $y = \sqrt{4 - \frac{x^2}{4}},\ y = \frac{x^2}{4\sqrt{2}}$ 11) $y = -\sqrt{4 - x^2},\ x^2 + 3y = 0$ 12) $y = \left|x^2 - 4x + 3\right|,\ y = 3$ 13) $y = \left|x^2 - 4\left|x\right| + 3\right|,\ y = 0$ 14) $x = y,\ x = \frac{\sqrt{3}}{\sqrt{4 - y^2}}$	15) $x = \frac{2}{y^2},\ x = \frac{1}{\sqrt{8 - y^2}},\ y = \sqrt{2}\ \left(y \ge 0\right)$ 16) $y = (2 + \cos x)\sin x$, y $= 0,\ x = \frac{\pi}{2},\ x = \frac{3\pi}{2}$ 17) $y = x\sqrt{1 + x^2}$, y $= 0,\ x = 1$ 18) $y = \frac{\ln x}{2\sqrt{x}}$, y $= 0,\ x = 1,\ x = e$ 19) $y = \frac{\sqrt{1 + \ln x}}{x}$, y $= 0,\ x = 1,\ x = e$ 20) $y = 0,\ y = \ln x,\ x = 2,\ x = e$ 21) $y = \frac{1}{\sin^2 x},\ y = \frac{1}{\cos^2 x},\ x = \frac{\pi}{6},\ x = \frac{\pi}{3}$ 22) $y = x^2,\ y = 4x^2,\ y = 4$ 23) $y = x(x+1)(x-2),\ y = 0,\ x = -2,\ x = 2$ 24) $y = xe^x,\ y = 0,\ x = -1,\ x = 2$ 25) $y^2 = 4x,\ x - y + 1 = 0,\ y = 0$ 26) $x - y^3 + 1 = 0,\ x + y - 1 = 0,\ y = 0$

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Bài giải

1) $S = \displaystyle\int_0^{2\pi} \left|\sin x\right| dx = \left|\int_0^{\pi} \sin x dx\right| + \left|\int_{\pi}^{2\pi} \sin x dx\right| = \left|-\cos x\Big|_0^{\pi}\right| + \left|-\cos x\Big|_{\pi}^{2\pi}\right|$ = 4 (đvdt).

2) $S = \displaystyle\int_{-1}^{2} \left|x^3\right| dx = \left|\int_{-1}^{0} x^3 dx\right| + \left|\int_0^2 x^3 dx\right| = \left|\frac{x^4}{4}\Big|_{-1}^{0}\right| + \left|\frac{x^4}{4}\Big|_0^2\right| = \frac{17}{4}$ (đvdt).

3) $x^2 - 2x = -x^2 + 4x \Leftrightarrow x = 0 \vee x = 3$

$\Rightarrow S = \displaystyle\int_0^3 \left|(x^2 - 2x) - (-x^2 + 4x)\right| dx = \left|\int_0^3 (2x^2 - 6x) dx\right| = \left|\left(\frac{2x^3}{3} - 3x^2\right)\Big|_0^3\right|$ = 9(đvdt).

4) $x^3 - 4x = 0 \Leftrightarrow x = 0 \vee x = 2 \vee x = -2$ (loại).

$$\Rightarrow S=\int_{-1}^{2}\left|x^3-4x\right|dx=\left|\int_{-1}^{0}(x^3-4x)dx\right|+\left|\int_{0}^{2}(x^3-4x)dx\right|=\left|\left(\frac{x^4}{4}-2x^2\right)\Bigg|_{-1}^{0}\right|+\left|\left(\frac{x^4}{4}-2x^2\right)\Bigg|_{0}^{2}\right|.$$

Vậy $S=\frac{23}{4}$ (đvdt).

5) $x^2-6x+5=0\Leftrightarrow x=1\vee x=5$ (loại).

$$\Rightarrow S=\int_{0}^{1}\left|x^2-6x+5\right|dx=\left|\int_{0}^{1}(x^2-6x+5)dx\right|=\left|\left(\frac{x^3}{3}-3x^2+5x\right)\Bigg|_{0}^{1}\right|.$$

Vậy $S=\frac{7}{3}$ (đvdt).

6) $x^2-3x+2=0\Leftrightarrow x=1\vee x=2$.

$$\Rightarrow S=\int_{0}^{2}\left|x^2-3x+2\right|dx=\left|\int_{0}^{1}(x^2-3x+2)dx\right|+\left|\int_{1}^{2}(x^2-3x+2)dx\right|$$

$$=\left|\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)\Bigg|_{0}^{1}\right|+\left|\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)\Bigg|_{1}^{2}\right|$$ = 1(đvdt).

7) $-x^2-2x=-x-2\Leftrightarrow x=-2\vee x=1$.

$$\Rightarrow S=\int_{-2}^{1}\left|x^2+x-2\right|dx=\left|\int_{-2}^{1}(x^2+x-2)dx\right|=\left|\left(\frac{x^3}{3}+\frac{x^2}{2}-2x\right)\Bigg|_{-2}^{1}\right|.$$

Vậy $S=\frac{9}{2}$ (đvdt).

8) $x^3-2x^2-x+2=0\Leftrightarrow x=2\vee x=\pm1$.

$$\Rightarrow S=\int_{-1}^{2}\left|x^3-2x^2-x+2\right|dx=\left|\int_{-1}^{1}(x^3-2x^2-x+2)dx\right|+\left|\int_{1}^{2}(x^3-2x^2-x+2)dx\right|$$

$$=\left|\left(\frac{x^4}{4}-\frac{2x^3}{3}-\frac{x^2}{2}+2x\right)\Bigg|_{-1}^{1}\right|+\left|\left(\frac{x^4}{4}-\frac{2x^3}{3}-\frac{x^2}{2}+2x\right)\Bigg|_{1}^{2}\right|.$$

Vậy $S=\frac{37}{12}$ (đvdt).

9) $\left|x\right|^3-2x^2-\left|x\right|+2=0 \Leftrightarrow \begin{cases} t=\left|x\right|\ge 0 \\ t^3-2t^2-t+2=0 \end{cases} \Leftrightarrow \begin{cases} t=\left|x\right|\ge 0 \\ \left[\begin{array}{l} t=1 \\ t=2 \end{array}\right. \end{cases} \Leftrightarrow \left[\begin{array}{l} x=\pm 1 \\ x=\pm 2 \end{array}\right.$.

$$\Rightarrow S=\int_{-2}^{2}\left|\left|x\right|^3-2x^2-\left|x\right|+2\right|dx=2\int_{0}^{2}\left|x^3-2x^2-x+2\right|dx$$

$$=2\left|\int_0^1 (x^3-2x^2-x+2)dx\right|+2\left|\int_1^2 (x^3-2x^2-x+2)dx\right|$$

$$=2\left|\left(\frac{x^4}{4}-\frac{2x^3}{3}-\frac{x^2}{2}+2x\right)\Bigg|_0^1\right|+2\left|\left(\frac{x^4}{4}-\frac{2x^3}{3}-\frac{x^2}{2}+2x\right)\Bigg|_1^2\right| = 3\text{(đvdt)}.$$

10) $\sqrt{4-\dfrac{x^2}{4}}=\dfrac{x^2}{4\sqrt{2}} \Leftrightarrow x^4+8x^2-128=0 \Leftrightarrow x=\pm 2\sqrt{2}$

$$\Rightarrow S=\int_{-2\sqrt{2}}^{2\sqrt{2}}\left|\sqrt{4-\frac{x^2}{4}}-\frac{x^2}{4\sqrt{2}}\right|dx=\left|\int_{-2\sqrt{2}}^{2\sqrt{2}}\left(\sqrt{4-\frac{x^2}{4}}-\frac{x^2}{4\sqrt{2}}\right)dx\right|$$

$$=2\left|\int_0^{2\sqrt{2}}\left(\sqrt{4-\frac{x^2}{4}}-\frac{x^2}{4\sqrt{2}}\right)dx\right|=\left|\int_0^{2\sqrt{2}}\sqrt{16-x^2}\,dx-\frac{1}{2\sqrt{2}}\int_0^{2\sqrt{2}}x^2dx\right|$$

$$=\left|16\int_0^{\frac{\pi}{4}}\cos^2 t\,dt-\frac{1}{2\sqrt{2}}\int_0^{2\sqrt{2}}x^2dx\right|=\left|8\left(t+\frac{1}{2}\sin 2t\right)\Bigg|_0^{\frac{\pi}{4}}-\frac{1}{2\sqrt{2}}\frac{x^3}{3}\Bigg|_0^{2\sqrt{2}}\right|.$$

Vậy $S=2\pi+\dfrac{4}{3}$ (đvdt).

11) $x^2+3y=0 \Leftrightarrow y=-\dfrac{x^2}{3} \Rightarrow -\sqrt{4-x^2}=-\dfrac{x^2}{3} \Leftrightarrow x^4+9x^2-36=0 \Leftrightarrow x=\pm\sqrt{3}$

$$\Rightarrow S=\int_{-\sqrt{3}}^{\sqrt{3}}\left|\sqrt{4-x^2}-\frac{x^2}{3}\right|dx=2\left|\int_0^{\sqrt{3}}\left(\sqrt{4-x^2}-\frac{x^2}{3}\right)dx\right|$$

$$=2\left|\int_0^{\sqrt{3}}\sqrt{4-x^2}\,dx-\frac{1}{3}\int_0^{\sqrt{3}}x^2dx\right|=2\left|4\int_0^{\frac{\pi}{3}}\cos^2 t\,dt-\frac{1}{3}\int_0^{\sqrt{3}}x^2dx\right|=2\left|2\left(t+\frac{1}{2}\sin 2t\right)\Bigg|_0^{\frac{\pi}{3}}-\frac{x^3}{9}\Bigg|_0^{\sqrt{3}}\right|.$$

Vậy $S = \dfrac{4\pi + \sqrt{3}}{3}$ (đvdt).

12) $\left|x^2 - 4x + 3\right| = 3 \Leftrightarrow \left[\begin{array}{l} x^2 - 4x + 3 = 3 \\ x^2 - 4x + 3 = -3 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 0 \\ x = 4 \end{array}\right.$.

Bảng xét dấu

x	0		1		3		4
$x^2 - 4x + 3$		+	0	–	0	+	

$$\Rightarrow S = \int_0^4 \left|\left|x^2 - 4x + 3\right| - 3\right| dx = \left|\int_0^1 \left(x^2 - 4x\right) dx\right| + \left|\int_1^3 \left(-x^2 + 4x - 6\right) dx\right| + \left|\int_3^4 \left(x^2 - 4x\right) dx\right|$$

$$= \left|\left(\frac{x^3}{3} - 2x^2\right)\Bigg|_0^1\right| + \left|\left(\frac{-x^3}{3} + 2x^2 - 6x\right)\Bigg|_1^3\right| + \left|\left(\frac{x^3}{3} - 2x^2\right)\Bigg|_3^4\right| = 8\text{(đvdt)}.$$

13) $\left|x^2 - 4\left|x\right| + 3\right| = 0 \Leftrightarrow x^2 - 4\left|x\right| + 3 = 0 \Leftrightarrow \left[\begin{array}{l} \left|x\right| = 1 \\ \left|x\right| = 3 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \pm 1 \\ x = \pm 3 \end{array}\right.$.

Bảng xét dấu

x	0		1		3
$x^2 - 4x + 3$		+	0	–	0

$$\Rightarrow S = \int_{-3}^3 \left|x^2 - 4\left|x\right| + 3\right| dx = 2\int_0^3 \left|x^2 - 4x + 3\right| dx$$

$$= 2\left[\int_0^1 \left(x^2 - 4x + 3\right) dx - \int_1^3 \left(x^2 - 4x + 3\right) dx\right]$$

$$= 2\left[\left(\frac{x^3}{3} - 2x^2 + 3x\right)\Bigg|_0^1 - \left(\frac{x^3}{3} - 2x^2 + 3x\right)\Bigg|_1^3\right].$$

Vậy $S = \dfrac{16}{3}$ (đvdt).

14) Tung độ giao điểm $y = \dfrac{\sqrt{3}}{\sqrt{4 - y^2}}$, $0 \le y < 2 \Leftrightarrow \left[\begin{array}{l} y = 1 \\ y = \sqrt{3} \end{array}\right.$

$$\Rightarrow S = \int_1^{\sqrt{3}} \left| \frac{\sqrt{3}}{\sqrt{4-y^2}} - y \right| dy = \left| \int_1^{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{4-y^2}} - y \right) dy \right| = \ldots$$

Vậy $S = 1 - \frac{\pi\sqrt{3}}{6}$ (đvdt).

15) Tung độ giao điểm $\frac{2}{y^2} = \frac{1}{\sqrt{8-y^2}} \Leftrightarrow y = 2$

$$\Rightarrow S = \int_{\sqrt{2}}^{2} \left| \frac{2}{y^2} - \frac{1}{\sqrt{8-y^2}} \right| dy = \left| \int_{\sqrt{2}}^{2} \left(\frac{2}{y^2} - \frac{1}{\sqrt{8-y^2}} \right) dy \right| = \ldots$$

Vậy $S = \sqrt{2} - 1 - \frac{\pi}{12}$ (đvdt).

16) $S = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left| (2+\cos x)\sin x \right| dx = \int_{\frac{\pi}{2}}^{\pi} (2+\cos x)\sin x dx - \int_{\pi}^{\frac{3\pi}{2}} (2+\cos x)\sin x dx$

$$= -\left(2\cos x + \frac{1}{4}\cos 2x\right)\Bigg|_{\frac{\pi}{2}}^{\pi} + \left(2\cos x + \frac{1}{4}\cos 2x\right)\Bigg|_{\pi}^{\frac{3\pi}{2}} = 3\text{(đvdt)}.$$

17) Hoành độ giao điểm $x\sqrt{1+x^2} = 0 \Leftrightarrow x = 0$

$$\Rightarrow S = \int_0^1 \left| x\sqrt{1+x^2} \right| dx = \int_0^1 x\sqrt{1+x^2} dx = \frac{1}{2}\int_0^1 \sqrt{1+x^2} d(1+x^2) = \frac{1}{3}\sqrt{(1+x^2)^3}\Bigg|_0^1.$$

Vậy $S = \frac{2\sqrt{2}-1}{3}$ (đvdt).

18) $S = \int_1^e \left| \frac{\ln x}{2\sqrt{x}} \right| dx = \int_1^e \frac{\ln x}{2\sqrt{x}} dx \left(\frac{\ln x}{2\sqrt{x}} > 0 \ \forall x \in [1;\ e] \right).$

Đặt $t = \ln x \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$x = 1 \Rightarrow t = 0,\ x = e \Rightarrow t = 1$

$$\Rightarrow S = \int_0^1 \frac{te^t dt}{2\sqrt{e^t}} = \int_0^1 t d\left(\sqrt{e^t}\right) = t\sqrt{e^t}\Big|_0^1 - \int_0^1 \sqrt{e^t} dt = \sqrt{e} - 2\sqrt{e^t}\Big|_0^1.$$

Vậy $S = 2 - \sqrt{e}$ (đvdt).

19) $S=\int\limits_1^e \left|\frac{\sqrt{1+\ln x}}{x}\right|dx=\int\limits_1^e \frac{\sqrt{1+\ln x}}{x}dx$.

Đặt $t=\sqrt{1+\ln x}\Rightarrow t^2=1+\ln x\Rightarrow 2tdt=\frac{dx}{x}$

$x=1\Rightarrow t=1,\ x=e\Rightarrow t=\sqrt{2}$

$\Rightarrow S=\int\limits_1^{\sqrt{2}} t.2tdt=\int\limits_1^{\sqrt{2}} 2t^2dt=\frac{2}{3}t^3\Big|_1^{\sqrt{2}}$.

Vậy $S=\frac{4\sqrt{2}-2}{3}$ (đvdt).

20) $S=\int\limits_2^e \left|\ln x\right|dx=\int\limits_2^e \ln xdx=x\ln x\Big|_2^e-\int\limits_2^e dx$.

Vậy $S=2-2\ln 2$.

21) $\frac{1}{\cos^2 x}=\frac{1}{\sin^2 x}\Leftrightarrow x=\frac{\pi}{4}\in\left[\frac{\pi}{6};\ \frac{\pi}{3}\right]$

$\Rightarrow S=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left|\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right|dx=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left|\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right|dx+\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left|\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right|dx$

$=\left|\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left(\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right)dx\right|+\left|\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left(\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right)dx\right|$

$=\left|(tgx+cotgx)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}\right|+\left|(tgx+cotgx)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right|$.

Vậy $S=\frac{8\sqrt{3}-12}{3}$ (đvdt).

22) Tọa độ giao điểm $\begin{cases}y=x^2\\y=4x^2\end{cases}\Leftrightarrow\begin{cases}x=0\\y=0\end{cases}$

Ta có: $\begin{cases}y=x^2\\y=4x^2\end{cases}\Leftrightarrow\begin{cases}x=\sqrt{y}\\x=\frac{1}{2}\sqrt{y}\end{cases}$

$$\Rightarrow S=\left|\int_0^4\left(\sqrt{y}-\frac{1}{2}\sqrt{y}\right)dy\right|=\left.\frac{\sqrt{y^3}}{3}\right|_0^4.$$

Vậy $S=\frac{8}{3}$ (đvdt).

23) $S=\int_{-2}^{2}\left|x(x+1)(x-2)\right|dx$

$$=\left|\int_{-2}^{-1}\left(x^3-x^2-2x\right)dx\right|+\left|\int_{-1}^{0}\left(x^3-x^2-2x\right)dx\right|+\left|\int_{0}^{2}\left(x^3-x^2-2x\right)dx\right|$$

$$=\left|\left.\left(\frac{x^4}{4}-\frac{x^3}{3}-x^2\right)\right|_{-2}^{-1}\right|+\left|\left.\left(\frac{x^4}{4}-\frac{x^3}{3}-x^2\right)\right|_{-1}^{0}\right|+\left|\left.\left(\frac{x^4}{4}-\frac{x^3}{3}-x^2\right)\right|_{0}^{2}\right|.$$

Vậy $S=\frac{37}{6}$ (đvdt).

24) $S=\int_{-1}^{2}\left|xe^x\right|dx=\int_0^2 xe^xdx-\int_{-1}^0 xe^xdx=\left.\left(x-1\right)e^x\right|_0^2-\left.\left(x-1\right)e^x\right|_{-1}^0.$

Vậy $S=\frac{e^3+2e-2}{e}$ (đvdt).

25) $\begin{cases}y^2=4x\\x-y+1=0\end{cases}\Leftrightarrow\begin{cases}x=\frac{1}{4}y^2\\x=y-1\end{cases}\Rightarrow\frac{1}{4}y^2=y-1\Leftrightarrow y=2$

$$\Rightarrow S=\int_0^2\left|\frac{1}{4}y^2-(y-1)\right|dy=\frac{1}{4}\left|\int_0^2\left(y^2-4y+4\right)dy\right|=\left.\frac{1}{4}\left(\frac{y^3}{3}-2y^2+4y\right)\right|_0^2.$$

Vậy $S=\frac{2}{3}$ (đvdt).

26) $\begin{cases}x-y^3+1=0\\x+y-1=0\end{cases}\Leftrightarrow\begin{cases}x=y^3-1\\x=1-y\end{cases}\Rightarrow y^3-1=1-y\Leftrightarrow y^3+y-2=0\Leftrightarrow y=1$

$$\Rightarrow S=\left|\int_0^1\left(y^3+y-2\right)dy\right|=\left|\left.\left(\frac{1}{4}y^4+\frac{1}{2}y^2-2y\right)\right|_0^1\right|.$$

Vậy $S=\frac{5}{4}$.

HT 16. Tính thể tích do hình phẳng giới hạn bởi các đường

1) $y = 3x,\ y = x\,,\ x = 0,\ x = 1$ quay quanh Ox	6) ellipse $(E): \dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ quay quanh Ox
2) $y = \dfrac{x^2}{2},\ y = 2\,,\ y = 4,\ x = 0$ quay quanh Oy	7) ellipse $(E): \dfrac{x^2}{16} + \dfrac{x^2}{9} = 1$ quay quanh Oy
3) $y^2 = (x-1)^3,\ x = 2$ và $y = 0$ quay quanh Ox	8) $y = x^2 + 2,\ y = 4 - x^2$ quay quanh Ox
4) $y^2 = 4 - x,\ x = 0$ quay quanh Oy	9) $y = x^2,\ y = \sqrt{x}$ quay quanh Ox
5) $(C): x^2 + (y-4)^2 = 4$ quay quanh Oy	10) $y = -\sqrt{4 - x^2},\ x^2 + 3y = 0$ quay quanh Ox

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Bài giải

1) $V = \pi\displaystyle\int_0^1 \left|(3x)^2 - x^2\right| dx = 8\pi\int_0^1 x^2 dx = \left.\frac{8\pi x^3}{3}\right|_0^1.$

Vậy $V = \dfrac{8\pi}{3}$ (đvtt).

2) Ta có $y = \dfrac{x^2}{2} \Leftrightarrow x^2 = 2y \Rightarrow V = \pi\displaystyle\int_2^4 x^2 dy = \pi\int_2^4 2y dy = \pi\, y^2\Big|_2^4.$

Vậy $V = 12\pi$ (đvtt).

3) Ta có $(x-1)^3 = 0 \Leftrightarrow x = 1 \Rightarrow V = \pi\displaystyle\int_1^2 y^2 dx = \pi\int_1^2 (x-1)^3 dx = \pi\left.\frac{(x-1)^4}{4}\right|_1^2.$

Vậy $V = \dfrac{\pi}{4}$ (đvtt).

4) Ta có $\begin{cases} y^2 = 4 - x \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} x = 4 - y^2 \\ x = 0 \end{cases} \Rightarrow y = \pm 2$

$\Rightarrow V = \pi\displaystyle\int_{-2}^{2} \left(4 - y^2\right)^2 dy = 2\pi\left.\left(16y - \frac{8y^3}{3} + \frac{y^5}{5}\right)\right|_0^2.$

Vậy $V = \dfrac{512\pi}{15}$ (đvtt).

5) Tung độ giao điểm $(C): x^2 + (y-4)^2 = 4$ và Oy:

$$(y-4)^2 = 4 \Leftrightarrow \left[\begin{array}{l} y - 4 = 2 \\ y - 4 = -2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} y = 6 \\ y = 2 \end{array}\right.$$

$$\Rightarrow V=\pi\int_2^6 x^2dy=\pi\int_2^6\left[4-(y-4)^2\right]dy=\pi\left(-\frac{y^3}{3}+4y^2-12y\right)\Bigg|_2^6.$$

Cách khác:

Hình khối tròn xoay là hình cầu bán kính R = 2 nên $V=\frac{4\pi 2^3}{3}$. Vậy $V=\frac{32\pi}{3}$ (đvtt).

6) Hoành độ giao điểm $(E):\frac{x^2}{16}+\frac{y^2}{9}=1$ và Ox là $x=\pm 4$.

Ta có: $\frac{x^2}{16}+\frac{y^2}{9}=1\Leftrightarrow y^2=\frac{9}{16}\left(16-x^2\right)$

$$\Rightarrow V=\pi\int_{-4}^4 y^2dx=\frac{9\pi}{16}\int_{-4}^4(16-x^2)dx=\frac{9\pi}{8}\left(16x-\frac{x^3}{3}\right)\Bigg|_0^4.$$

Vậy $V=48\pi$ (đvtt).

7) Tung độ giao điểm $(E):\frac{x^2}{16}+\frac{y^2}{9}=1$ và Oy là $y=\pm 3$.

$$\frac{x^2}{16}+\frac{y^2}{9}=1\Leftrightarrow x^2=\frac{16}{9}\left(9-y^2\right)$$

$$\Rightarrow V=\pi\int_{-4}^4 x^2dy=\frac{16\pi}{9}\int_{-3}^3(9-y^2)dy=\frac{32\pi}{9}\left(9y-\frac{y^3}{3}\right)\Bigg|_0^3.$$

Vậy $V=64\pi$ (đvtt).

8) Hoành độ giao điểm $x^2+2=4-x^2\Leftrightarrow x=\pm 1$

$$\Rightarrow V=\pi\int_{-1}^1\left|\left(x^2+2\right)^2-\left(4-x^2\right)^2\right|dx=24\pi\int_0^1\left|x^2-1\right|dx=24\pi\left|\left(\frac{x^3}{3}-x\right)\Bigg|_0^1\right|.$$

Vậy $V=16\pi$ (đvtt).

9) Hoành độ giao điểm $x^2=\sqrt{x}\Leftrightarrow x^4=x\Leftrightarrow x=0\vee x=1$

$$\Rightarrow V=\pi\int_0^1\left|x^4-x\right|dx=\pi\left|\int_0^1\left(x^4-x\right)dx\right|=\pi\left|\left(\frac{x^5}{5}-\frac{x^2}{2}\right)\Bigg|_0^1\right|.$$

Vậy $V=\frac{3\pi}{10}$ (đvtt).

10) Hoành độ giao điểm $-\sqrt{4-x^2} = -\dfrac{x^2}{3} \Leftrightarrow x^2 = 3 \Leftrightarrow x = \pm\sqrt{3}$

$$\Rightarrow V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} \left|\left(4-x^2\right) - \frac{x^4}{9}\right| dx = \frac{2\pi}{9}\left|\int_0^{\sqrt{3}} \left(36 - 3x^3 - x^4\right) dx\right| = \frac{2\pi}{9}\left|\left(36x - 3x^3 - \frac{x^5}{5}\right)\Big|_0^{\sqrt{3}}\right|.$$

Vậy $V = \dfrac{28\pi\sqrt{3}}{5}$ (đvtt).

Xin chân thành cảm ơn quý thầy cô và các bạn học sinh đã đọc tài liệu này!

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